December 2015

Multi-Dimensional Assessment of Transit System Efficiency and Incentive-based Subsidy Allocation

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MULTI-DIMENSIONAL ASSESSMENT OF TRANSIT SYSTEM EFFICIENCY AND INCENTIVE-BASED SUBSIDY ALLOCATION

by

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A Dissertation Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

in Engineering

at

The University of Wisconsin-Milwaukee

December 2015

ABSTRACT

MULTI-DIMENSIONAL ASSESSMENT OF TRANSIT SYSTEM EFFICIENCY AND INCENTIVE-BASED SUBSIDY ALLOCATION

by

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The University of Wisconsin-Milwaukee, 2015
Under the Supervision of Professor Yue Liu

Over the past several decades, contending with traffic congestion and air pollution has emerged as one of the imperative issues across the world. Development of a transit-oriented urban transport system has been realized by an increasing number of countries and administrations as one of the most effective strategies for mitigating congestion and pollution problems. Despite the rapid development of public transportation system, doubts regarding the efficiency of the system and financing sustainability have arisen. Significant amount of public resources have been invested into public transport; however complaints about low service quality and unreliable transit system performance have increasingly arisen from all walks of life. Evaluating transit operational efficiency from various levels and designing incentive-based mechanisms to allocate limited subsidies/resources have become one of the most imperative challenges faced by responsible authorities to sustain the public transport system development and improve its performance and levels of service.

After a comprehensive review of existing literature, this dissertation aims to develop a multi-dimensional framework composed of a series of robust multi-criteria evaluation models to assess the operational and financial performance of transit systems at various levels of application (i.e. region/city level, operator level, and route level). It further contributes to bridging the gap between transit efficiency evaluation and the subsequent subsidy allocation by developing a set of incentive-based resource allocation models taking various levels of operational and financial efficiencies into consideration. Case studies using real-world transit data will be performed to validate the performance and applicability of the proposed models.

To

my parents,

and my wife

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ACKNOWLEDGMENTS

I would like to express my gratefulness to my supervisor, Prof. Yue Liu for all his support and encouragement throughout my whole Ph.D time. I deeply appreciate the opportunity he gave me to work with him. He has been always helpful, supportive, and offering constructive advices, through all the phases of my studies. He has been an example not only on the way of working and conducting research but also in thinking in general.

Special thanks also go to my committee members, Prof. Jie Yu, Prof. Xiao Qin, Prof. Lingfeng Wang and Prof. Lingqian Hu for their valuable contributions to my dissertation.

I also want to thank all the people that have been in the same research group for these years and made the office a pleasant environment. Thanks everyone, Peng Li, Yaojun Wang and Yun Yuan.

Last but not the least, I would like to dedicate this dissertation to my loving family, Dad, Mom and my wife, Judy Chen. With your consistent help and loving, I am brave enough to face the difficulties.

I love you all.

Chapter 1: Introduction

1.1. Background

Over the past several decades, contending with traffic congestion and air pollution has emerged as one of the imperative issues across the world. Development of a transit-oriented urban transport system has been realized by an increasing number of countries as one of the most effective strategies for mitigating congestion and pollution problems.

According to the 2011 statistics of public transport in United States, the total number of transit trips reached 10,319 million while the total number of passenger mileages was 56,077 million kilometers. The total operational expense on the transit system was up to 38,362.1 million dollars. In the meanwhile, the capital investment on public transport has already reached 17,057.1 million dollars. The transit industry has produced more than 380,000 jobs with 13,557.6 million dollars fare collection (2013 Public Transport Fact Book, 2013). In developing economies, for example in China, the total amount of passengers by transit has reached 77.9 billion till the end of 2009, within which 74.3 billion are by bus transit, and 3.6 billion are by rail transit. The total length of public transit lines has reached 289,000 kilometers, while the total length of bus lanes has reached 7,452 kilometers; urban rail transit systems have been operated in 12 cities in mainland, with the total number of 64 routes and 1291 stations, and the total length of 1556.2 kilometers. Bus Rapid Transit (BRT) system has been developed to over 10 cities.

Despite the rapid development of public transportation system, doubts regarding the efficiency of the system and financing sustainability have arisen. In US, according to the 2010 version of MTC report, the transit in Bay Area of the Unite States has been allocated approximately $1.5 billion subsidies to compensate its operation loss and maintain its service level. Since 1983, transit has received a share of the federal user fees paid by drivers, principally through fuel taxes. Additional diversions from federal user fees have been authorized by the Congestion Mitigation and Air Quality Improvement (CMAQ) program. In 2010, the latest year for which data are available, the total diversion from federal user fees approached $6 billion (Federal Transit Administration, National Transit 2010 Database, 2012). In China, from 2007 to 2013, the annual revenue of Beijing's rail transit rose from 1.2 billion yuan ($200 million) to 3.2 billion yuan ($530 million), while the operation expenditure increased from 1.3 billion ($220 million) yuan to 6.7 billion yuan ($1100 million). In 2012, the Beijing government subsidized its buses with 15 billion yuan ($2500 million) (China Daily, 7/8/2014). In the past six years, the Beijing government has totally allocated more than 95 billion yuan ($16 billion) subsidies to its public transport. Shanghai public transport has received more than 3.5 billion yuan ($600 million) from government in 2013 which was used to compensate operational loss and maintain facilities. Significant amount of public resources have been invested into public transport; however complaints about low service quality and unreliable transit system performance have increasingly arisen from all walks of life. Therefore, evaluating transit operational efficiency from various levels and accordingly allocating limited subsidy/resources have become the most imperative challenges faced by many responsible authorities to sustain the public transport system development and to improve its performance and levels of service.

To contend with this vital issue, studies focused on various levels of transit performance evaluation or efficiency assessment have been proposed in the literature over the past several decades (Nathanail, 2008; Tyrinopoulos and Antoniou, 2008; Eboli and Mazzulla, 2007, 2009, 2011; Hassan et al., 2013; Badami and Haider, 2007; Lao and Liu, 2009; Boile, 2001; Zhu, 2003; Karlaftis, 2004; Nakanishi and Falcocchi, 2004; Tsamboulas, 2006; Barnum et al., 2008; Sheth et al., 2007; Lao and Liu, 2009; Sanchez, 2009; Yu and Fan, 2009; Zhao et al., 2011; Hawas et al., 2012; Karlaftis and Tsamboulas, 2012; Sheth et al., 2007; Abreha, 2007). Certainly, those research efforts have made invaluable contributions to evaluation of transit performance from different perspectives. However, much remains to be advanced on the development of a multi-dimensional transit system efficiency assessment framework. In addition, existing mathematical models and methodologies may not be sufficient for transit system evaluation and deserve further extension and enhancement.

In transit resource/subsidy allocation, the most existing studies (Jolliffe and Hutchinson, 1975; Bowman and Turnquist, 1981; Zahavi 1979, 1982; Douglas, 1998; Cervero, 1998; Bhatta and Drennan, 2003) have attempted to use traditional capital-based or cost-proportional (e.g. total mileage, fuel consumption, or total passenger-trips) methods to allocate subsidies to transit operators to cover their operational loss and encourage them to provide better services in the next operational cycle. Those capital/cost-based methods, though effective to keep financial stability of transit operators, may not actually function to provide sufficient incentives for them to improve their performance. Many studies have indicated that there exists a negative correlation between the amount of capital-based subsidy and a transit operator's performance (Obeng and Sakano, 2008). This is due to the fact that the operational performance or efficiency of transit operators has not been properly integrated into the subsidy allocation process. In other words, the higher the loss/cost a transit operator the higher the subsidy it would be compensated. In review of relevant literature, very few studies have linked efficiency evaluation with the subsidy allocation, resulting in lack of effective framework and methodology for incentive-based subsidy allocation.

In view of all such importance of the public transportation system and the complexities that often exist in its evaluation and resource allocation process, development of a comprehensive framework for transit system assessment for various levels of application and accordingly design incentive-based subsidy allocation mechanisms remain challenging.

1.2. Research Objectives

The primary objective of this dissertation is to develop an integrated framework with quantitative approaches for comprehensive multi-dimensional transit system efficiency assessment and incentive-based subsidy allocation. More specifically, this research contributes to:

- Developing robust multi-criteria evaluation models for transit system efficiency assessment at various levels of application;
- Designing a framework and operational mechanisms to integrate transit efficiency evaluation with transit subsidy allocation;
- Developing theoretically justified and practically applicable models for incentive-based transit subsidy allocation; and
- Applying developed models and operational procedures to real-world cases, and provide guidelines to public transportation authorities.

1.3. Dissertation Organization

Based on the proposed research objectives, this study proposes to organize the primary research tasks into six chapters. The core of those tasks and their interrelations are illustrated in Figure 1.1.

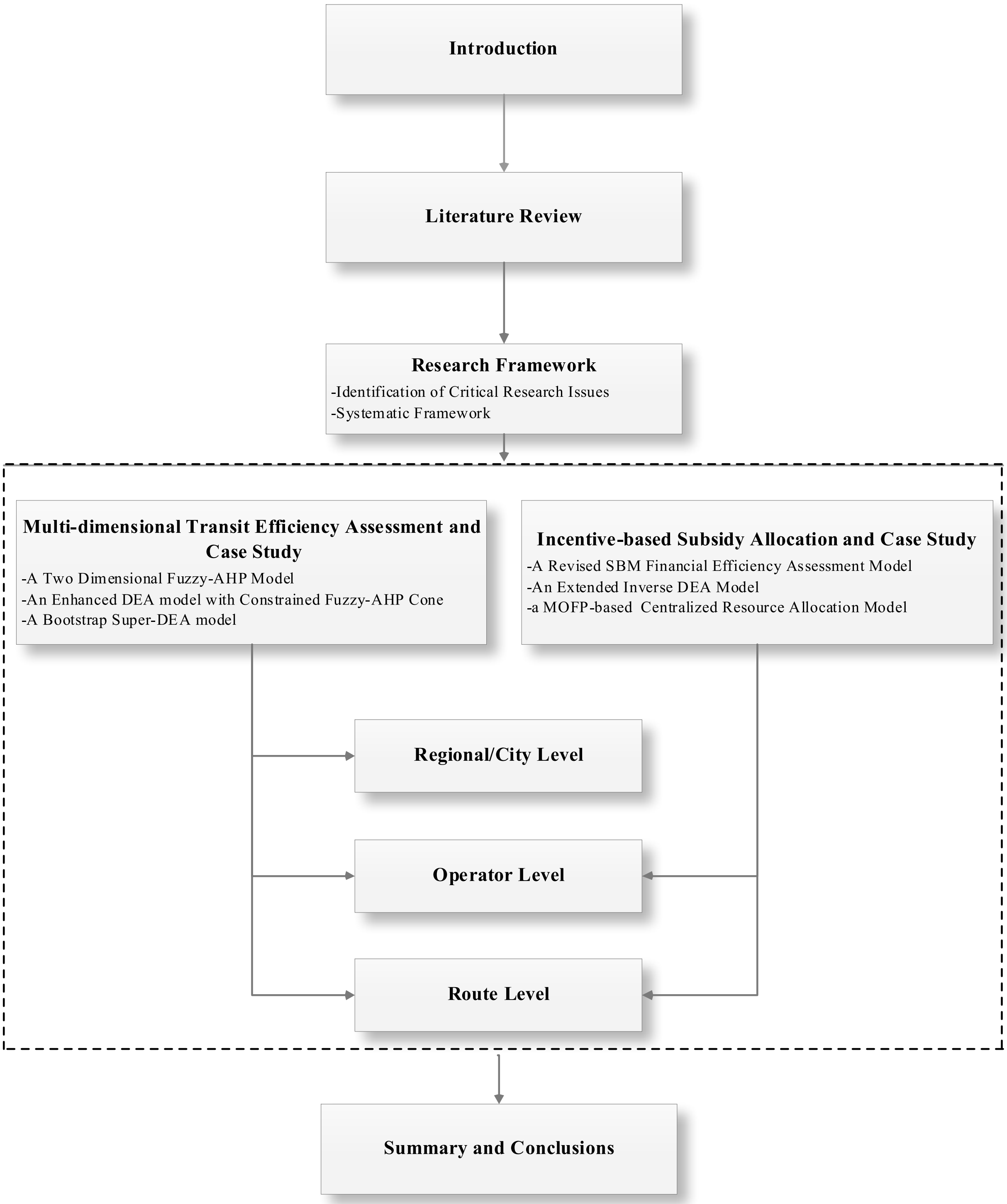


Figure 1.1 Dissertation organization

The remaining chapters of this dissertation are proposed to be organized as follows:

- **Chapter 2** presents a comprehensive literature review of existing studies on various dimensions of transit system efficiency assessment and performance evaluation, as well as transit subsidy allocation, including both research methodologies and applications. The review focuses on identifying the advantages and limitations of those studies, along with their potential enhancements.
- **Chapter 3** illustrates the proposed research framework, based on critical issues that need to be taken into consideration in the development of the multi-dimensional transit efficiency assessment and incentive-based subsidy allocation models. It specifies the key modules and their functional features in this framework.
- **Chapter 4** proposes to develop formulations of models for transit performance and efficiency evaluation at three different levels of application, namely the city/regional level, the operator level and the route level. **At the city/regional level**, this study develops a framework with both policy and technical layers, which offers the advantage of preventing the vagueness and uncertainty when evaluating technical criteria while properly retaining the policy preferences from decision makers. The policy layer is designed to better capture a city's characteristics and developing priorities as well as the subjective opinions of various transit stakeholders, based on which technical criteria are further compared and assessed in the "technical layer" with an innovative fuzzy Analytical Hierarchy Process (AHP) model, where a non-linear optimization formulation is proposed to maximize the consistency in pair-wise comparison and weight estimation. **At the operator level**, an enhanced DEA (data envelopment analysis) model with constrained cones is proposed to offer advantages

in considering weights assigned to different performance indicators when the efficiency of transit operators are assessed. Such modeling improvement can remedy the deficiency of traditional DEA models in evaluating the relative efficiency of decision-making units but not allowing for ranking of the efficient units themselves. **At the route level**, this study proposes to develop a Bootstrap Super-DEA model based on empirical data and repeated sampling to prevent the errors due to imperfect data and judgment mistakes. The proposed model is also expected to yield the confidence intervals to suggest the efficiency boundaries for each bus route operation. Case studies will be performed for all three levels of models.

- **Chapter 5** will develop the incentive-based subsidy allocation mechanisms for bus operators and bus routes. **At bus operator-level**, a revised slacks-based measures of super efficiency model for financial efficiency assessment is proposed to examine the financial performance of transit operators, which will be then integrated with the operational efficiency obtained from Chapter 4 using an inverse DEA model for subsidy allocation. To test the sensitivity and reliability of the proposed model, the chapter presents the results of extensive analyses with a real-world case in Chongqing Metropolis, China. **At bus route-level,** a Multi-objective Fractional Programming-based model is constructed to assign Bus Company's incentive-based subsidy as well as distributing Company's targets into to its managed bus routes simultaneously. As a natural extension of bus operator-level subsidy allocation study, 17 bus routes in Chongqing Third Bus Company are selected as a case study to share the incentive-based subsidy and set targets of ridership and mileage increases.

- **Chapter 6** summarizes the contributions of this dissertation and the directions for future research.

Chapter 2: Literature Review

2.1. Introduction

In view of the large body of literature on various aspects of public transportation research, this chapter will present a comprehensive review of only those research efforts in transit efficiency assessment, performance evaluation, and resource/subsidy allocation. The purpose is to identify the special characteristics, strengths, and deficiencies of existing studies and thus to define the primary directions for this study.

2.2. Transit Efficiency Assessment

Efficiency, a concept originated in industrial engineering, describes the relation of inputs to outputs, and is concerned with minimizing inputs for a specific output or maximizing output for a specific input. The Development Assistance Committee of Organization for Economic Co-operation and Development (OECD) defines efficiency in terms of transformation of inputs into results. Similarly, welfare economists sometimes define efficiency based on the transformation of costs into benefits as measured, for example, by benefit-cost ratios. In both cases, efficiency assessment is defined by how economically inputs or costs are transformed into results or benefits.

Efficiency assessment is very commonly used in many areas to evaluate a unit or system's performance and to further target their weakness. For example, Song et al. (2013) analyzed and compared the energy efficiency among BRICS (Brazil, Russia, India, China and South Africa) to realize that energy efficiency of BRICS as a whole is low but has a quickly increasing trend. Shrivastava et al. (2010) reviewed the relative technical efficiency of 60 coal fired power plants in India by using CCR and BCC models of data envelopment analysis. In addition to efficiency evaluation, target benchmark of input variables has also been evaluated. Halkos and Tzeremes (2013) evaluated the Top 25 European Football Club's efficiency levels in order to analyze how European football clubs' current value and debt levels influence their performance. Lin et al. (2010) implemented the economic performance assessment to local government in China to evaluate and rank all alternatives. Phillip and Lee (2013) examined energy efficiency in the Japanese transportation sector and then unfold comparisons with the United States and other developed economies. Coate (1999) described an efficiency approach to the evaluation of policy changes by comparing it with other possible changes which might be made from the status quo. Huang et al. (2011) introduced a dynamic two-stage approach to analyzing the hotel industry's technical efficiency at the sub-national level. Victor and Raquel (2011) used a subjective performance evaluation method to help organization ensure equal opportunities for men and women. Chen and Yan (2011) constructed an alternative network DEA model that embodies the internal structure for supply chain performance evaluation.

Transit efficiency assessment, as an application of general efficiency evaluation methodologies, aims to evaluate how well a transit system utilizes available labor and capital resources (Gilbert and Dajani, 1975; Fielding et al., 1978; Fielding et al., 1985; Chu et al., 1992; Nolan, 1996; Karlaftis, 2003) to provide quality services. Efficiency assessment has become an essential task for transit service providers to capture passenger demand trends, operational constraints, concerns of stakeholders, and changing service needs. It also allows the responsible authorities to achieve better economic performance assessment, organization administration, and transit planning and financing.

In review of literature, previous research on transit efficiency assessment focuses primarily on the transit service side, falling into three different categories, namely the user perception/satisfaction based approach, the efficiency indicator based approach, and the integrated approach with user opinions and efficiency indicators both considered (Hassan et al., 2013).

User perception/satisfaction based approach examines transit performance by the transit users' perception or satisfaction (Nathanail, 2008; Tyrinopoulos and Antoniou, 2008; Eboli and Mazzulla, 2007; 2009; 2011), where the different aspects of the transit service are rated by the users by a satisfaction survey. The most commonly used indicators include reliability, frequency, capacity, fare, cleanliness, comfort, security, staff, information, and the ticketing system. Efficiency indicator based approach features the use of various variables of relevance to the transit system demand and operation such as loading/ridership, travel time, travel distance, frequency, service duration, revenue, manpower, cost, accident data, fuel consumption and emission to calculate the ''efficiency'' indicators (Badami and Haider, 2007; Lao and Liu, 2009). Efficiency indicators are primarily needed to quantify the productivity of the system components (vehicles, route and operation), cost, environment, and safety. In addition, other studies introduced the concepts of transit availability or transit service accessibility as the indicators to measure efficiency, where the spatial elements and social economic factors, such as service coverage, service span and service population, are normally taken into accounts (Polzin et al., 2002; Rood, 1997; Hillman, 1997). Very recent research has started to integrate users' opinions and efficiency indicators into a unified framework (Sheth et al., 2007; Abreha, 2007). Those service-oriented performance evaluation methods, though perfect capturing transit user opinions, lack consideration of other aspects of transit system, e.g. infrastructure and safety, which has limited their applicability in comprehensive evaluation of transit system development.

To direct and monitor transit system performance and promote public transport development, many countries and municipalities have developed guidelines and standards for transit efficiency or performance assessment. For examples, in 1984 the U.S. Department of Transportation (USDOT) published a synthesis on bus service evaluation methods to review and provide supplemental materials for use by the transit industry. In 1995, a synthesis of transit practice on ten projects funded by the Transit Cooperative Research Program (TCRP) was published, where a survey of transit agencies in North America indicates that as many as 44 different evaluation criteria were used in the transit industry. The selected criteria covered activities related to bus route design and operation, ranging from location of bus stops to the hours of service in the area of route level service delivery (Kittelson & Associates, Inc. et al., 2003). In 2003, The Transit Capacity and Quality of Service Manual (TRB, 2003) has developed guidelines for evaluating the performance of public transport system. The manual has categorized the evaluation index system into three groups which are station, route, and system. Moreover, all the three groups are required to be ranked in terms of accessibility and convenience that are decided by the indicators of frequency, occupancy, services hours, punctuality and the time gap between private car and public transport.

In European countries, the International Association of Public Transport (UTIP) has set up a group of indictors including the population of transit users, the services coverage, the number of bus routes, stations, vehicles, the vehicle mileage, the patronage, the average trip distance and the fare to compare the performance of public transport system among the different cities and regions (UTIP, 2011). In addition, the European Committee for Standardization (CEN) EN 13816 standard was produced with the aim of promoting a quality approach to public transport operations, and to focus interest on the needs and expectations of customers. Areas of measurement based on the categories in the European Standard EN 13816 were integrated into the Key Performance Indicator (KPI) system.

In developing countries such as China, despite the rapid development of public transportation system in China in recent years, it is still at its beginning stage considering the percentage of urban public transit investment in the nation's GDP (1% in 2006 for China vs. 3%-5% for developed countries).To contend with the accelerated urbanization process, the Chinese government has planned to aggressively invest in the public transportation system in the future. One of the most important programs is to support 30 Chinese cities to develop the "Transit Metropolis" before 2015 according to the "12th five-year Transport Development Plan" released by the Ministry of Transport of China. This program is expected to elevate the urban public transportation system performance and completeness and significantly improve the transit ridership in urban transportation system. The Ministry of Transport of China has developed a series of technical criteria (e.g. public transit share, coverage of transit stations, transfer times, subsidy, energy efficiency, safety) to select the candidates to be transit metropolis. On the other hand, different cities may vary in their priorities of public transportation system development policies. The combinational impact of various technical criteria and policy priorities is expected to determine the level of public transportation system development in a region. Although various kinds of standards from both national prospective and industry prospective could be found to guide transit development, most of them focus on developing evaluation criteria system, there lacks a theoretically justified and practically applicable framework as well as the robust models for convenient evaluation and comparison of transit system developing levels.

From the perspective of application, substantial efforts have been made to develop various methods and models to evaluate and compare system-level, operator-level and route-level's transit performance or efficiency.

At the system level, index measures are normally employed to produce a single value to reflect the combined and weighted result covering various kinds of transit activities. For examples, Horowitz and Thompson (1995) have constructed a list of 70 generic objectives for evaluation of an intermodal transfer facility after extensive literature review and interviews with various stakeholders. Nolan (1996) has conducted a study of 25 mid-sized bus agencies using USDOT section 15 data from 1989 to 1993, and tried to identify the relationships between the efficiency scores and agency characteristics using To-bit regression. Fu and Xin (2002) have proposed a new performance index called Transit Service Indicator (TSI), which could be used as a comprehensive measure to evaluate the quality of transit system. Their framework took into account spatial and temporal variations in travel demand and recognized that quality of service is a result of interaction between supply and demand. Tsamboulas (2006) has assessed the performance of 15 European transit systems, in terms of efficiency and effectiveness. Furthermore, efforts have been made towards identifying the sources of inefficiency, and determining whether the new modes of transport industry including competition and/or private ownership have actually led to "improved" transport service provision. Xu and Lian (2011) have proposed an evaluation system, including convenience, adaptability, and efficiency which was further divided into eleven indictors to assess the performance of the transit system.

The majority of studies on the system-level transit evaluation were designed for developed regions with relatively mature transit systems. There lacks of sufficient attention on developing areas with many other critical factors (e.g. infrastructure and fuel consumption) taken into consideration. In addition, many multi-criteria evaluation methods using in those studies lack sufficient flexibility in altering their evaluation framework to account for the interaction between the importance of technical criteria and the preferences of decision makers.

For efficiency evaluation at the operator level, Gathon (1989) presented a study of efficiency evaluation of urban transit firms. In his study, ordinary least squares and free disposal hull approaches were utilized to compare the performance of 60 firms across European countries, where the number of seat-kilometers traveled was used as the output measure and the labor hours of work was the input measure. Chu et al. (1992) have developed a single index for measuring service efficiency as well as service effectiveness of public transit agencies. The authors argued that measures of efficiency, which were based on service production, should be treated separately from measures of effectiveness, which were based on service consumption. Kerstens (1996) has evaluated and compared the performance of French urban transit companies using a broad selection of nonparametric reference technologies for two specifications of the production process. Yeh et al. (1999) have presented an effective fuzzy multi-criteria analysis (MA) approach to performance evaluation for urban public transport companies in Taiwan involving multiple criteria of multilevel hierarchies and subjective assessments of decision alternatives. Parkan (2002) has carried out a study to obtain comprehensive performance ratings to gauge the productive and service quality performance of a public transit company using a recent performance measurement method called operational competitiveness rating (OCRA) analysis. In his study, the computing ratings incorporate the cost and revenue efficiency of operations, quality of service experience as perceived by commuters, and quality of service delivery in specific areas measured internally. De Borger et al. (2002) have conducted an extensive review and analysis of the literature on the production and cost frontiers for public transit operators.

Their paper summarized many critical issues, including technical versus scale versus allocative efficiencies, the selection of input and output measures, returns to scale and scope, and the impact of ownership and government subsidies. Othman and Mahmod (2010) proposed a multi criteria decision making in ranking the bus companies using fuzzy rule, and a corresponding numerical case study was given to prove the model. Hahn et al. (2012) have developed a network Data Envelopment Analysis (DEA) model for evaluating the efficiency of bus companies of Seoul, Korea which successfully took environment issues into account. The model can reflect the non-storable nature of public transportation services by sequentially considering transportation services provided by operators and consumed by users.

At the route level, Boile (2001) has developed a procedure to identify both technical and scale efficiencies for a selected group of bus transit lines. Karlaftis (2004) used data envelopment analysis and globally efficient frontier production functions to investigate two important issues in transit line operation efficiency: 1) the relationship between efficiency and effectiveness and 2) the relationship between performance and scale economies. Sheth et al. (2007) unfolded a study of performance evaluation of bus routes from the perspectives of both operators and passengers, and the provision of bus services along different routes that comprise a public transit network is assessed. Lao and Liu (2009) proposed a model integrating GIS to compute each bus line's operational efficiency and spatial effectiveness scores. This approach allows for close inspection and comparison of operational and spatial aspects of bus lines. Similarly, Hawas et al. (2012) have developed a GIS-based model to evaluate the baseline performance level of Al Ain Public Bus Service in United Arab Emirates (UAE) according to some selected input (travel time per round trip, total number of stops, total number of operators, total number of buses) and output (daily ridership and vehicle-kilometer) variables.

In terms of evaluation methodologies, multi-criteria ranking methods are generally used for performance analysis and evaluation. Many studies have been proposed focusing on the combination of fuzzy logic model with multi objective decision that can assist in reducing judgment errors (Yamashita, 1997; Turban et al., 2000; Yeh et al. 2000; Hanaoka and Kunadhamraks 2009; Campos et al. 2009; Yu et al. 2011; Zak et al. 2011; Hassan et al., 2013).

Hassan et al., (2013) has selected and further modified Technique of Order Preference by Similarity to Ideal Solution (TOPSIS) as their multi criteria evaluation method to assess transit service performance, where an enhanced weighting process was presented to determine the weight for criteria and indicator in a generalized transit system, as following:

$$W_{jk}^{I} = \frac{\sum_{l=1}^{L}\left(W_{jkl}^{I}\right)}{L} \quad \forall j,k,l\ j = 1,\dots,J, k = 1,\dots,K$$

$$W_{j}^{C} = \frac{\sum_{l=1}^{L}\left(W_{jl}^{C}\right)}{L} \quad \forall j\ \ j = 1,\dots,J$$

$$\frac{\sum_{k=1}^{K}\left(W_{jkl}^{I}\right)}{L} = 1\ \forall j,l,j = 1,\dots,J,l = 1,\dots,L$$

$$\frac{\sum_{j=1}^{J}\left(W_{jl}^{C}\right)}{L} = 1\ \forall l,l = 1,\dots,L$$

$$W_{jk}^{Cl} = W_{j}^{C} \times W_{jk}^{l}$$

Where r denote the route index of the N routes in the system, $r = 1,\dots,N$ These N routes are to be evaluated based on a set of J criteria, which are independent to each other. Let j define the index of criterion, $j = 1,\dots,J$. Each criterion, C_j, is divided into K indicators. Let k define the index of indicator, $k = 1,\dots,K$. Each indicator k of criterionj, I_{jk}, represents some specific quantitative measure of performance. A group of L experts are asked to provide separate weights for each criterion, C_j, and indicator,I_k, where l is the index of the expert, $l = 1,\dots,L$.

Yeh et al. (2000) has developed a fuzzy AHP (Analytic Hierarchy Process) framework to evaluate the performance of Taiwan bus companies by integrating both the fuzzy–analytical hierarchy process and the fuzzy–multi criteria decision-making. The proposed model features in defining a triangular fuzzy membership functions, as follows:

$$\mu_{\forall}(x) = \begin{Bmatrix} 0, & x_i \leq a_1 \\ x_i - a_1 / T - a_1 & a_1 \leq x_i \leq T \\ a_3 - x_i / a_3 - T & T \leq x_i \leq a_3 \end{Bmatrix}$$

$$f_{ij} = \left\{ \frac{\mu f_{ij}(x)}{x}, x \in X \right\}$$

where x_i is fuzzy evaluation of alternative in term of triangular fuzzy number, T is the vertex of the triangular fuzzy number and *a1* and *a3* are the two endpoints, f_{ij} is the fuzzy set membership of subjective evaluation mark (i = 1, 2, …n, alternatives and j = 1, 2, …m, the criteria environment), $\mu f_{ij}(x)$ is the fuzzy set score of average fuzzy performance rating of alternatives according to criteria.

When assessing the efficiency of operators or bus routes, most existing studies assume transit units as production lines, and evaluate the efficiency of such lines by comparing multiple inputs and outputs (Fare and Grosskopf, 1996, 2000; Seiford and Zhu, 1999; Boile, 2001; Nolan et al. 2002; Sexton and Lewis, 2003; Zhu, 2003; Karlaftis, 2004; Nakanishi and Falcocchi, 2004; Hwang and Kao, 2006; Tsamboulas, 2006; Barnum et al., 2008; Kao and Hwang, 2008; Sheth et al., 2007; Lao and Liu, 2009; Sanchez, 2009; Yu and Fan, 2009; Zhao et al., 2011; Hawas et al., 2012; Karlaftis and Tsamboulas, 2012). In this regard, Data Envelopment Analysis (DEA), a non-parametric method introduced by Farrell (1957) and popularized by Charnes et al. (1978), is usually the first-choice by the majority of researchers. DEA is a managerial approach to assess relative performance/efficiency for evaluating decision making units (DMUs). Each DMU selects its best set of weights corresponding to consider inputs and outputs; the values of weights may thus vary from one DMU to another. The DEA models then calculate each DMU's performance score ranging between zero and one that represents its relative degree of efficiency (Wei and Chang, 2011).

Initially, many researchers (Fare and Grosskopf, 1996, 2000; Seiford and Zhu, 1999; Boile, 2001; Nolan et al. 2002) adopted conventional DEA model to assess the performance or efficiency of transit units. The selected classical BCC model is illustrated as below:

$$Max_{u,v} \quad \theta_k = \frac{\sum_{m=1}^{M} u_m y_{mk}}{\sum_{n=1}^{N} v_n x_{nk}}$$

$$s.t. \qquad \frac{\sum_{m=1}^{M} u_m y_{mj}}{\sum_{n=1}^{N} v_n x_{nj}} \leq 1 \; \forall j$$

$$\sum_{n=1}^{N} v_n x_{nk} = 1$$

$$v_n, u_m, y_{mj}, x_{nj} > 0 \;\; \forall j, m, n$$

Where j is an index of decision making unit (DMU), *j=1… J*, *n* is an index of input, *n*=1… *N*, *m* is an index of output, *m*=1… *M*, X_{nj} is the n^{th} input for the j^{th} DMU, Y_{mj} is the m^{th} input for the j^{th} DMU, u_m, v_n are two non-negative scalars (weights) for the m^{th} output and the n^{th} input, and θ_k is the efficiency/effectiveness ratio of DMU_k.

Review of the literature indicates that application of the DEA in transit efficiency evaluation has several limitations. For example, DEA is unable to further distinguish efficient units and the reliability of evaluation results could be potentially degraded by unrepresentative data sample. In addition, DEA ignores the inevitable variation of efficiency of decision-making units. Most importantly, DEA calculations are traditionally value-free and the underlying assumption is that no output or input is more important than the other, although in the real world there often exists different importance over different input or output indicators (Halme et al., 1999). Neglect of this may result in biased evaluation results.

Due to those reviewed decencies, some other scholars have made a series of valuable attempts to modify and enhance the classical DEA model. Lao and Liu (2009) have integrated DEA model with geographic information systems, and then the model was employed to evaluate the performance of Monterey-Salinas Transit system. Higashimoto et al.,(2013) proposed a network DEA model to assess the bus routes efficiency in Tomakomai city. The formulation of the network DEA can be written as following:

$$\theta_0^* = min_{\lambda^k s^{k-}} \sum_{k=1}^{K} W^k \left[1 - \frac{1}{m_k} (\sum_{i=1}^{m_k} \frac{s_i^{k-}}{x_{i0}^k}) \right]$$

s.t. $x_0^k = X^k \lambda^k + s^{k-}$ $(k = 1, \dots, K)$

$$y_0^k = Y^k \lambda^k + s^{k+}\ (k = 1, \dots, K)$$

$$e\lambda^k = 1\ (k = 1, \dots, K)$$

$$\lambda^k \geq 0, s^{k-} \geq 0, s^{k+} \geq 0, (\forall k)$$

$$Z^{(k,h)} \lambda^h = Z^{(k,h)} \lambda^k, \left(\forall (k,h)\right)$$

$$Z_0^{(k,h)} = Z^{(k,h)} \lambda^k, \left(\forall (k,h)\right)$$

$$Z_0^{(k,h)} = Z^{(k,h)} \lambda^h, \left(\forall (k,h)\right)$$

$$\sum_{k=1}^{K} W^k = 1, W^k \geq 0 (\forall k)$$

Jorda et al., (2012) used the super-DEA model developed by Andersen and Petersen (1993) analyze the technical efficiency of bus services in Span. The model is descripted as following:

$$Min\ \theta - \varepsilon\left(\sum_{j}^{n} s_j^- + \sum_{j}^{n} s_j^+\right)$$

$$s.t.\sum_{j=1, j\neq j_0}^{n} \lambda_j\, x_j + s_j^- = \theta x_{j0}$$

$$\sum_{j=1, j\neq j_0}^{n} \lambda_j\, y_j - s_j^+ = y_{j0}$$

$$\lambda_j, s_j^-, s_j^+ \geq 0$$

where x_j is an m-dimensional input vector and y_j is an s-dimensional output vector for the j0th unit; s_j^- is an m-dimensional slack variable vector for input variables while s_j^+ is an s-dimensional slack variable vector for output variables; θ is a scalar defining the share of the j0th DMU input vector which is required in order to produce the j0th DMU output vector within the reference technology; λ is an intensity vector in which λ_j denotes the intensity of the j0th unit; ε is a non-Archimedean infinitesimal.

2.3. Subsidy Allocation

Subsidy is a direct or indirect payment, economic concession, or privilege granted by a government to private firms, households, or other governmental units in order to promote a public objective. Subsidy allocation falls into the category of general resource allocation. Unlike the concept of efficiency assessment, resource allocation derives from business investment which aims to find out the best option to fully utilize limited resources in an investment decision. Consequently, the resource allocation is a method of indicating the sort of projects which are most likely to fit the available resources and a simple procedure which will assist in assessing the relative merits of these projects (Pearson, 1967).

Resource allocation has been attracting ever-increasing attentions from researchers because of a remarkable role in determining success or failure of a project. Calinescu et al. (2013) have addressed the problem of resource allocation in survey designs and discuss its impact on the quality of the survey results. They propose a novel method in which the optimal allocation of survey resources is determined such that the quality of survey results, i.e., the survey response rate, is maximized. Amirteimoori and Emrouznejad (2012) have proposed a DEA-based model to determine an optimal input/output resource allocation plan for banking sector with limitation in IT investment. Sadeghi and Ameli (2012) have presented an analytical hierarchy process (AHP) decision model for sectoral allocation of energy subsidy based on several criteria. Many attempts are made to uses the Data Envelopment Analysis (DEA) technique to solve the problem of allocating a fixed cost across a set of comparable decision making units (DMUs) in a fair way (Amirteimoori and Kordrostami, 2005; Lin, 2011; Amirteimoori and Emrouznejad, 2011; Bi et al., 2011). Wiseman (2014) has provided an overview of Mooney's contributions to the use of community values in priority setting and resource allocation in health care. McCarthy et al. (2010) have proposed a non-linear optimization model incorporating with uncertainty to allocate resource for efficient environmental management. Their study showed that the theory solved a diverse range of important problems of resource allocation, including distributing conservation resources among the world's biodiversity hotspots. Konur et al. (2013) have proposed a mathematic modelling approach to resource allocation for railroad-highway crossing safety upgrades.

Transit system, receiving a substantial part of financial support from government, is faced travelers' discontent over what they perceive as an inefficient, ineffective, and unaccountable public transport services under conditions of slowed economic growth, demand for more and better service, and general cost escalation. Consequently, how to efficiently allocate subsidy to transit system is raising ever-increasing research interests.

To finance public transport, government expects to benefit the community as a whole as well as achieving welfare maximization. Bhatta and Drennan (2003) have found considerable evidence that public transit yields benefits to the community in the form of increased output, increased productivity, lower production costs, higher incomes, higher property values, higher employment, and reduced noncommercial travel time. The important role of transit systems to society has led that almost all the public transit agencies receive a considerable portion of their operating income from taxpayer dollars. Virtually no transit agency could exist without heavy public subsidies. Regarding US case, in 2002 alone, transit providers nationally received about $12.8 billion in capital funds from various sources, with 41% from the federal government, 12% from state sources, 20% from local sources, and the remainder from taxes levied by transit agencies and other directly generated sources (American Public Transportation Association, 2005). Due to a heavy financial burden to governments, the subsequent doubts about whether it is a drain on system assets requiring inordinate amounts of attention, finances, and scarce resources are naturally followed. Meanwhile, although the mounting public resources are invested into public transport, the complaints about lower service quality and unreliable transit system increasingly arises from all walks of life. The issues have caused a debate on the requisite need to efficiently and reasonably allocate the subsidy into transit system.

A thorough review of literature indicates that most of relevant studies on transit subsidy allocation are traditional capital-based or cost-proportional methods (e.g. based on or proportional to total mileage, fuel consumption, or total passenger-trips) to compensate transit operators to cover their operational loss and preserve their financial stability. In those models, there are two commonly used criteria to allocate transit subsidies, namely equity and economic efficiency (Douglas, 1998). Equity is always evaluated in terms of the ability-to-pay principle, in which users should contribute to the cost of services according to their income ability (Cervero, 1998). The efficiency focuses on economist's efficiency arguments such as economics of scale and external benefits. Consequently, users are required to pay for the cost of services in line with the benefits they receive (Douglas, 1998). The way of allocating subsidy considering "equity" is to offset all of a proportion of the difference between service fare and service cost. One of commonly used approaches following the equity criterion is the expenditure-income ratio method, which was developed by Zahavi (1979). On the other hand, when considering efficiency, transit subsidy allocation is usually done via the benefit-cost ratio analysis, where the user cost plays an important role in subsidy allocation, which is believed to have impact on the transit services unreliability (Bowman and Turnquist, 1981; Jolliffe and Hutchinson, 1975). A handful of practical models have been proposed since the 1980s for transit subsidy allocation. For examples, Glaister and Lewis (1978) have developed a quantitative estimation model of public transit subsidies for London from the viewpoint of peak and non-peak passenger volume. Tisato et al. (1992) developed a subsidy calculating model based on the public transit service quality. These capital-based or cost-proportional methods, though effective to keep financial stability of transit operators, may not actually function to provide sufficient incentives for them to improve their performance in the next operational cycle. Many studies have indicated that there exists a negative correlation between the amount of capital-based subsidy and a transit operator's performance (Obeng and Sakano, 2008). This is due to the fact that the operational performance of transit operators has not been properly integrated into the subsidy allocation process. In other words, the higher the loss/cost a transit operator incurs the higher the subsidy it would be compensated.

In the past decade, the general public has become more demanding on the efficient utilization of limited public resources and expected higher service quality from the transit system. Such pressure leads to the increasing emphasis on the transition from the traditional cost-based transit subsidy allocation to the Performance-based Budgeting (PBB) system, in which the resources or subsidies are allocated according to transit system's performance.

The concept of PBB is not new to public administration. Schultz (2004) has described PBB as a type of public sector budgeting that uses information on the performance of an agency or program to help determine the level of resources allocated to it. The aim is to provide governments with information that allows them to determine how efficient and effective current activities are and whether better value for money can be achieved by changing the level or mix of resources allocated. Such a system was designed to enable budgeters and policymakers to make substantive budget choices, as traditional budgeting processes are no longer considered satisfactory. However, applying PBB into transit subsidy allocation remains challenging and is still at its exploratory stage although some states have made some attempts to establish their own PBB system to budget their transit systems. For example, Mandizvidza (2005) has unfolded an examination and analysis of the application of the performance-based budgeting systems in California urban transit agencies. However, the author mainly discussed the performance measurement system used to implement the PBB for transit agencies via a survey, and no methods have been proposed related to subsidy allocation. In review of literature, very limited efforts have been made to develop incentive-based or performance-based transit subsidy allocation models.

2.4. Summary

In summary, this chapter has provided a comprehensive review of existing research efforts in the transit system efficiency assessment and subsidy allocation. Limitations of previous studies have been identified to be used to constitute the basis for subsequent developments of the multi-dimensional transit system efficiency assessment and incentive-based subsidy allocation framework and models. Some additional areas which have not been adequately addressed in existing literature are summarized below:

- There lacks a multi-dimensional framework for transit efficiency evaluation for various levels of applications with both subjective judgments and objective assessment from multiple stakeholders taken into account;
- There lacks sufficient investigations in identifying indicators or criteria at various levels of applications;
- Most previous studies on transit efficiency/performance evaluation focus on the service and operational aspects, which can find their best application in developed regions with well-established transit systems. For areas that are still in the developing stage, the comprehensive impacts of other critical factors such as developing policies/priorities, infrastructure/facilities, energy/sustainability, and/or safety on urban transit system development have not been sufficiently investigated in previous studies;
- Most commonly used multiple criteria ranking methods, e.g. Analytic Hierarchy Process or Data Envelopment Analysis, lack sufficient flexibility in altering their evaluation framework to account for the interaction between the importance of technical criteria and the preferences of decision makers. They also fail to provide reliable ranking and assessment results when the dataset used is limited and

unrepresentative. In addition, how to prevent the very unbalanced scale, vagueness, and uncertainty of judgment when weighting the importance of different criteria remains challenging;

- Traditional capital-based subsidy allocation methods, though effective to keep financial stability of transit operators, may not actually function to provide sufficient incentives for them to improve their performance. There lacks an effective theoretical modeling framework in literature that can feed transit efficiency assessment into subsidy allocation in a close-loop way; and
- There lacks an overall operational framework or guidelines that can effectively integrate the efficiency assessment and subsidy allocation models for real-world application.
- Previous studies display an absence of route level incentive-based subsidy allocation and targets setting mechanisms with the applicable models.

In view of the above limitations in the existing studies, this research aims to develop a comprehensive and robust multi-dimensional transit system evaluation framework for various levels of applications. In the meantime, this research is expected to contribute to filling the vacancy of a theoretically justified and practically applicable model that can prioritize limited resources to urban transit operators according to their operational and financial efficiencies. Additionally, a route-level subsidy allocation and target setting model is also activated to bridge the gap in relevant research areas. Operational guidelines will be also developed and validated through extensive real-world case studies to assist responsible agencies in best application of the proposed models.

Chapter 3: A Systematic Modeling Framework

3.1. Introduction

This chapter will illustrate the modeling framework of the proposed research and the interrelations between its principle components. Also included are the key research issues in the development of each modeling component and proposed primary research tasks to address those issues.

3.2. Key Research Issues and Primary Research Tasks

Some major research issues to be addressed in this research are listed below:

- Selection of evaluation criteria or indicators for transit efficiency assessment at various levels of applications;
- Design of a multi-dimensional assessment framework, which coordinates interactions among key evaluation models and features the flexibility to alter the evaluation framework to account for the variation of the importance of evaluation criteria;
- Development of a set of multi-criteria transit efficiency assessment models for various levels of applications, which can provide reliable ranking and assessment results for limited and unrepresentative dataset and prevent the unbalanced scale, vagueness, and uncertainty of judgment when weighting the importance of different criteria;
- Development of a financial efficiency evaluation model to measure financial performance of transit operators during subsidy allocation;
- Design of an incentive-based transit subsidy allocation mechanism and models for bus operators, which take into account both operational efficiency and financial efficiency;

- Design of a route-level subsidy allocation and target setting models, which help authority to subdivide company's subsidy and targets into bus routes simultaneously; and
- Application of the proposed frameworks and models in real-world case studies to validate their applicability and provide guidelines to responsible agencies.

It should be noted that all above research issues are interrelated and each is indispensable for the proposed research. To address these critical issues, this proposal has divided the research efforts into the following research tasks falling into three major categories:

Framework design:

- **Task 1**: Develop a comprehensive multi-dimensional evaluation framework composed of multiple modules to perform city/region level, operator-level, and route level transit efficiency evaluation.
- **Task 2**: Develop an incentive-based subsidy allocation frameworks based on operational and financial efficiencies.
- **Task 3**: Develop an overall operational framework and guidelines that can effectively integrate the efficiency assessment and subsidy allocation models for real-world application.

Model development:

- **Task 4**: Develop a robust two-level multi-criteria evaluation model for city/regional level transit system efficiency evaluation, where a "policy level" is designed to better capture a city's characteristics and developing priorities as well as the subjective opinions of various transit stakeholders, based on which technical criteria are further compared and assessed in the "technical level" with an enhanced fuzzy Analytical

Hierarchy Process (AHP) model, where a non-linear optimization formulation is proposed to maximize the consistency in pair-wise comparison and weight estimation. The developed two-level framework offers the advantage of preventing the vagueness and uncertainty of the decision-maker(s) when evaluating technical criteria while properly retaining the policy preferences. The evaluation model is expected to generate macroscopic rankings of different cities in terms of transit system development and also to identify microscopic deficiencies and areas of improvement.

- **Task 5**: Develop an enhanced Data Envelopment Analysis (DEA) model with constrained cones to examine the efficiency of multiple transit operators with the preferences over various input and output indicators better captured.
- **Task 6**: Develop a Bootstrap Super-DEA approach to evaluate route-level bus operational efficiency based on empirical data and advanced repeated sampling approach to improve the estimation of the critical value precision statistics and fine-tune the evaluation results from small sample dataset. To prevent the errors due to imperfect data and judgment mistakes, an efficiency interval estimator will be also developed to suggest the efficiency boundaries for each bus route.
- **Task 7**: Estimate financial efficiency by comparing operational costs and operational dataset with a revised SBM super efficiency model.
- **Task 8**: Develop an innovative target-setting-based inverse DEA model to allocate the limited subsidies with the objectives to maintain financial sustainability and to improve operational efficiency.
- **Task 9**: Develop a centralized resource allocation model to subdivide bus operator's subsidy and targets into bus routes.

Case studies:

- **Task 10**: Use illustrative real-world examples to demonstrate the procedure of each proposed framework and further test each developed model.

3.3. Modeling Framework

In view of the above research tasks, Figure 3.1 depicts the framework of the proposed system for this dissertation, highlighting interrelations between principal system components. This study will focus only on those modules within the transit system efficiency assessment tools for different dimensions as well as incentive-based subsidy allocation model.

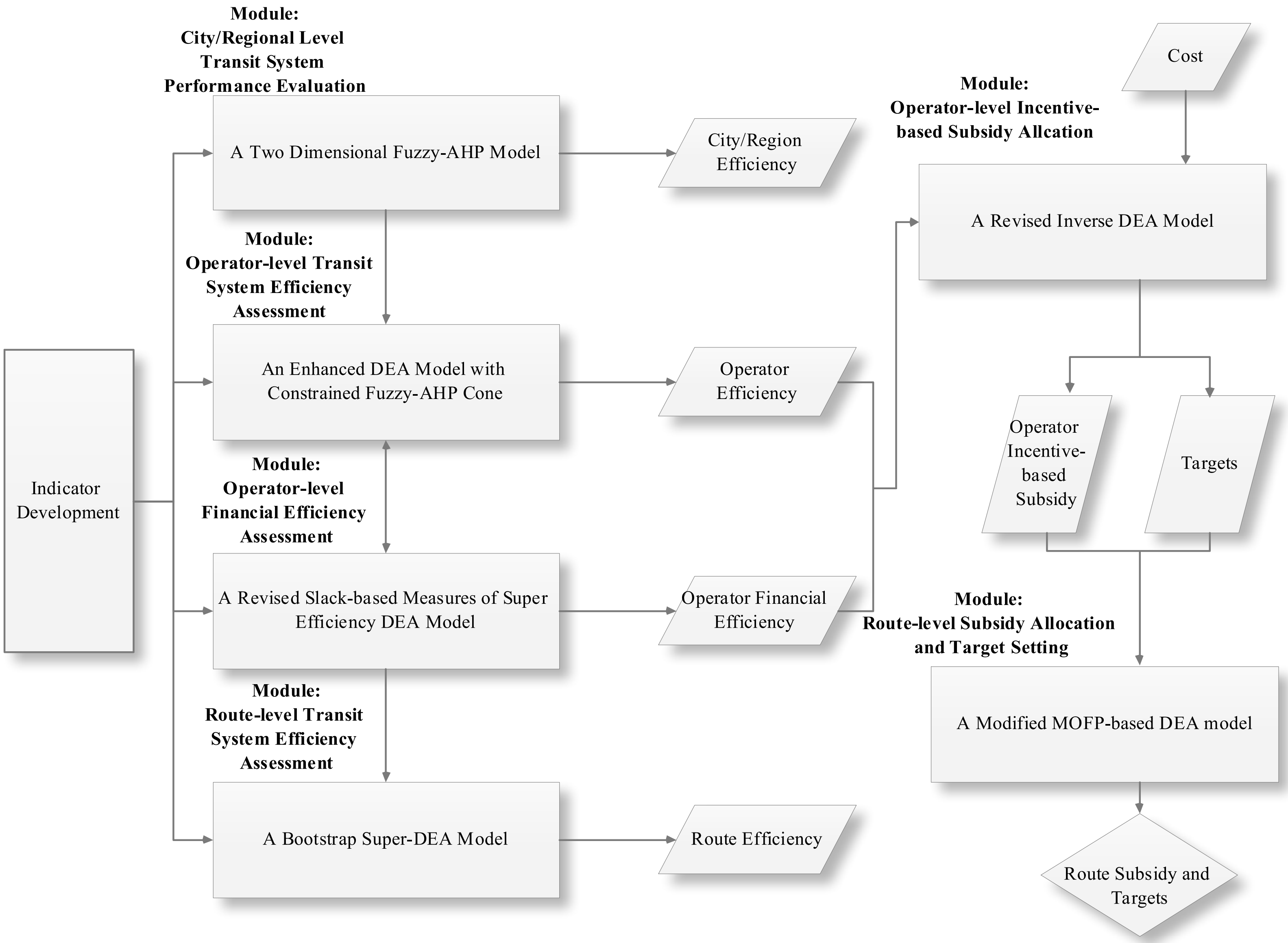


Figure 3.1 A modeling framework of the proposed dissertation

Chapter 4: Multi-Dimensional Transit Efficiency Assessment

4.1. Introduction

This chapter will propose to develop formulations of models for transit performance and efficiency evaluation at three different levels of applications, namely the city/regional level, the operator level and the route level. At the city/regional level, this study develops a framework with both policy and technical layers, which offers the advantage of preventing the vagueness and uncertainty when evaluating technical criteria while properly retaining the policy preferences from decision makers. The policy layer is designed to better capture a city's characteristics and developing priorities as well as the subjective opinions of various transit stakeholders, based on which technical criteria are further compared and assessed in the "technical layer" with an innovative fuzzy Analytical Hierarchy Process (AHP) model, where a non-linear optimization formulation is proposed to maximize the consistency in pair-wise comparison and weight estimation. At the operator level, an enhanced DEA (data envelopment analysis) model with constrained cones is proposed to offer advantages in considering weights assigned to different performance indicators when the efficiency of transit operators are assessed. Such modeling improvement can remedy the deficiency of traditional DEA models in evaluating the relative efficiency of decision-making units but not allowing for ranking of the efficient units themselves. At the route level, this study proposes to develop a Bootstrap Super-DEA model based on empirical data and repeated sampling to prevent the errors due to imperfect data and judgment mistakes. The proposed model is also expected to yield the confidence intervals to suggest the efficiency boundaries for each bus route operation. Case studies will be performed for all three levels of models.

4.2. City/Region-level Transit System Efficiency Assessment

4.2.1 Research Motivation

The basic aim and principle of urban public transport is to maximize its social services as well as keeping a financial sustainability. To conclude the features of city or regional public transport system development, there is a basic argument that the strategies for urban transit development are significantly influenced by national and local policies. While the solutions to promote transit system are often context-specific and every city will need to find ways to improve public transport based on detailed local needs and conditions. As a result, how to include and reflect policies' impacts turns to be a critical issue when evaluating city/regional transit system performance. Unfortunately, the literature review reveals that the existing studies rely on either purely subjective methods, likely, conventional AHP method, or complicate objective mathematical modeling. Consequently, development of a practicable framework and model to integrate subjective judgments and objective assessment is a demanding point of this study.

By review the methodology in performance evaluation of urban transit system, as a widely valuable method, AHP, a subjective method for multi-criteria decision-making process introduced by Saaty (1980), has been commonly used in transportation system evaluation studies(Zhang et al., 2002; Larson and Forman, 2007; Filippoet al., 2007; Wei et al., 2007).However, the following critical issues deserved further investigation during the application of AHP, which are: 1) how to handle the very unbalanced scale of judgment, 2)how to properly construct the pair-wise comparison matrix subject to the biased impacts from the objective judgment, selection and preference of decision-makers. In view of the literature, the most commonly used approach for constructing the pair-wise comparison matrix in the AHP is to rely on the knowledge of experts, which may sometimes result in arbitrary and biased decisions. In estimating the weights for all criteria, eigenvalue method (Saaty, 1980; Golden et al., 1989), logarithmic least squares method (Bryson, 1995; Yu, 2002), the geometric mean method (Sudhakar and Shrestha, 2003), and linear programming methods (Chandran et al., 2005; Wang et al., 2008) have all been widely used. However, due to the vagueness and uncertainty on judgments of decision makers, the crisp pair wise comparison by the aforementioned methods in the conventional AHP still remains insufficient and imprecise to capture the importance of different criteria.

Most importantly, the conventional AHP does not offer sufficient flexibility in altering the evaluation framework to account for the interaction between the importance of technical criteria and the preferences of decision makers. For example, the decision makers in a city with well-established transit system may focus more on the safety and economic performance while another city with relatively new system developing would pay more attention to the service and operations. Therefore, it is critical to take the decision makers' or experts' subjective opinions into consideration to better capture a city's characteristics and developing priorities as well as the concerns of various transit stakeholders, on the basis of which technical criteria can be further assessed and compared.

To remedy such limitations, the objective of this section is to develop an assessment framework with sufficient flexibility to capture all the contributory factors for evaluating and comparing the level of urban public transportation development. It will focus on the following critical research tasks:

- Categorize the evaluation criteria related to a broad range of transit planning and development concerns and develop a two-level evaluation framework to separate subjective judgments and objective assessment;

- Propose a robust model to tackle the multi-criteria evaluation problem, which features the integration of the fuzzy logic with a hierarchical AHP structure to: 1) normalize the scales of different evaluation indicators, 2) construct the matrix of pair-wise comparisons with fuzzy set, 3) optimize the weight of each criterion with a non-linear programming model, and 4) synthesize the final score for evaluating the transit development levels; and
- Illustrate the proposed framework and model through an illustrative example case to assist government and major municipalities in best understanding and applying the proposed model during the process of developing transit system.

4.2.2 The Evaluation Framework

The framework developed in this section features a structure with evaluation carried out at two levels (the policy level and the technical level), using multi-criteria evaluation approach at each level, and enabling the interaction between two levels through weight integration. Those weights either reflect the experts' preferences on policy priorities or capture the importance of various technical criteria.

4.2.2.1 The Policy Level

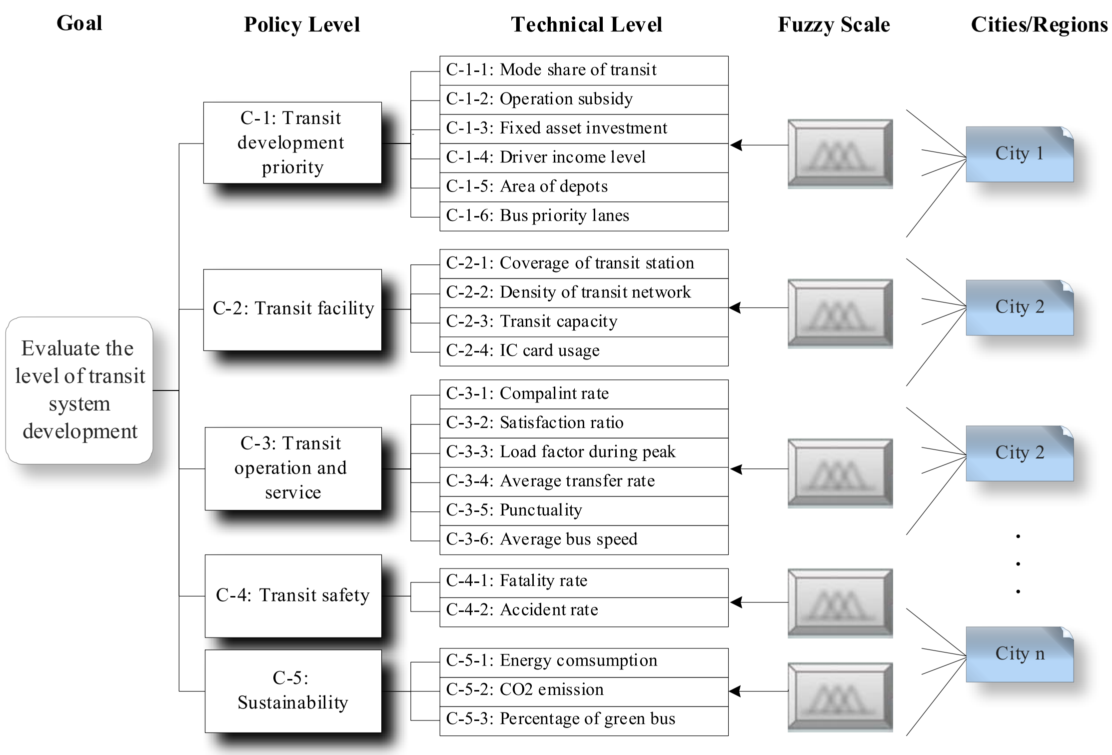
One unique feature of the developed framework in this paper is the involvement of the stakeholders and experts to share their knowledge, opinions and understanding of transit system development by introducing the policy level, at which five aspects related to transit development priority, transit facility construction, transit operation and service, transit safety, and sustainability are properly weighed into consideration. Considering the subjective nature of decisions made in this level, a simple weighting process that involves a set of decision makers and experts is used (Saaty, 1980).

4.2.2.2 The Technical Level

Given the preferences determined at the policy level, this study further expands the aforementioned five policy criteria into 21 technical criteria (see Figure 1 for details) that will be evaluated at the technical level with an enhanced fuzzy AHP model. A fuzzy scale level is added to facilitate the normalization of different criteria scales. Note that the weights determined for each criterion at the technical level will be further synthesized with weights obtained from the policy level to get the final ranking score for each alternative.

In summary, Fig. 4.1 outlines a graphical illustration of the proposed framework for transit system development evaluation, consisting of the following modules:

- Goal: The goal initially established by decision makers is to evaluate, compare and rank the level of transit system development for a set of alternative cities/regions;
- Policy level: A set of criteria capturing the stakeholders and experts' knowledge, opinions and understanding of transit system development;
- Technical level: A comprehensive list of evaluation criteria constitutes the second level of the hierarchy. Detailed descriptions for these criteria can be found in Section 2;
- Fuzzy scale: The fuzzy membership functions are employed to normalize the scales of different technical criteria so as to represent the satisfaction of each criterion with respect to each alternative; and
- Cities/Regions: A set of candidate cities/regions to be evaluated



C-1-1	Total volume of public transport trips / Total volume of the trips
C-1-2	Public transport operations subsidy by the local government / Total volume of public transport passengers
C-1-3	Investment in public transport's fixed assets / Investment in whole transport sector's fixed assets
C-1-4	Average monthly income of public transport drivers / Local average monthly income
C-1-5	Total areas of local bus depots and stations / Total number of the local bus vehicles
C-1-6	Total length of bus priority lanes / Total length of road network
C-2-1	(500 meter coverage area of bus stations+ 800 meter coverage of metro stations) / urban built-up area
C-2-2	Total length of public transport network / Urban built-up area
C-2-3	Total capacity of transit system / Local population*10000
C-2-4	Number of public transport trips that use IC cards/ Total number of public transport trips
C-3-1	Number of complaint cases / Total number of public transport trips
C-3-2	Number of surveyed satisfied passengers to the local transit system / Total number of surveyed passengers
C-3-3	Maximum section of passenger flow during the peak hour / Maximum section capacity during the peak hour
C-3-4	Number of trips completed by transferring / Total number of public transport trips
C-3-5	Number of buses sticking to the schedule at terminals / Total number of the buses
C-3-6	average bus speed
C-4-1	Number of public transport involved fatalities / the mileage*106
C-4-2	Number of public transport involved accidents / the mileage*106
C-5-1	Total energy consumption / Total turnover of public transport passengers
C-5-2	Total emission of CO2 / Total turnover of public transport passengers
C-5-3	Number of buses with qualified Euro IV emission standards / Total number of buses

Figure 4.1 The proposed evaluation framework

4.2.3 The Multi-Criteria Evaluation Model

At the policy level, to reflect the subjective opinions of experts and decision makers, a conventional AHP model with the eigenvalue method is adopted to obtain the weights of all policy criteria, given by: $\{w_i | i = 1, \dots, n\}$. Details about the conventional AHP model and the eigenvalue method which are not the focus of this paper can be found in Saaty (1980).

With the preferences of experts and decision makers taken into consideration, relative importance of technical criteria is further determined by an enhanced fuzzy AHP model to effectively prevent the vagueness and uncertainty on judgments. The advantages of the proposed fuzzy-AHP structure lie in its capability to: 1) normalize the scales of different technical indicators, 2) construct the matrix of pair-wise comparisons with fuzzy set, and3) optimize the weight of each criterion with a non-linear programming model to maximize the judgment consistency. Such advantages offer more subjective evaluation of various technical criteria and allow identification of deficiencies of transit system development with respect to a specific criterion. Fig. 4.2 summarizes the overall procedure of the evaluation model which will be detailed in the following sections.

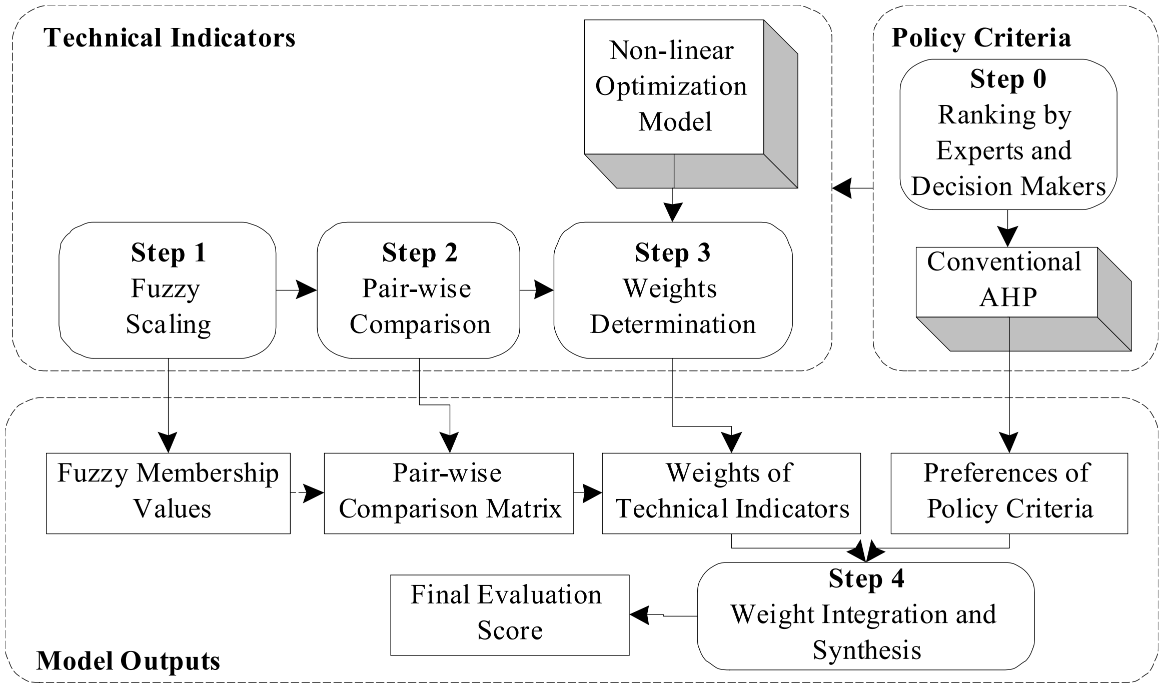


Figure 2.2 The overall evaluation procedure

To facilitate the model presentation, all definitions and notations used hereafter are summarized in Table 4.1.

Table 4.1 Notation of key parameters used in the proposed model

i	Index corresponding to policy criteria$(i = 1 \ldots n)$
j	Index corresponding to technical criteria$(j = 1 \ldots m_i)$ under policy criterion i
k	Index corresponding to cities to be evaluated $(k = 1 \ldots K)$
x_{jk}^{i}	Indicator representing the selected city k being evaluated by technical criterion j
μ_{jk}	Fuzzy membership value corresponding to indicatorx_{jk}^{i}
$\bar{\mu}_j^i$	Average fuzzy membership value for technical criterion j
$x_{j(min)}^{i}$	The minimal crisp value for technical criterion j
$x_{j(max)}^{i}$	The maximal crisp value for technical criterion j
s_j^i	Standard deviation of indicator values corresponding to technical criterion j
s_{min}^{i}	$min\{s_j^i \mid j = 1 \cdots m_i\}$
s_{max}^{i}	$\max\{s_j^i \mid j = 1 \cdots m_i\}$
$A_i = (a_{jl}^{i})_{m_i x m_i}$	Pair-wise comparison matrix for technical indicators under policy criterion i
a_{m_i}	Comparison scale for the pair-wise comparison matrix of technical indicators under policy criterion i
w_i	Weight for the policy criterion i
w_j^i	Weight for the technical criterion j under policy i
$Y_i = (y_{jl}^{i})_{m_i x m_i}$	Consistency judgment matrix for technical indicators under policy criterion i
$CIC(m_i)$	Consistency index coefficient for technical indicators under policy criterion i
s_k	The synthesized evaluation score of city k

4.2.3.1 Fuzzy Scaling

Under each policy criterion i, there are a set of technical criteria ($j = 1 \ldots m_i$) to be evaluated for each city. In view of the difficulty in comparing technical indicators with different types of units, this step has employed a set of fuzzy membership functions to normalize the scales of different technical indicators, based on the characteristics of each technical indicator. Two types of indicators, i.e., "the-lower-the-better" and "the-higher-the-better" are identified to normalize x_{jk}^i with their fuzzy sets, given by:

For the-lower-the-better indicators:

$$\mu_{jk}^i = \frac{\left[x_{j(max)}^i + x_{j(min)}^i - x_{jk}^i\right]}{\left[x_{j(max)}^i + x_{j(min)}^i\right]} \qquad (1)$$

For the-higher-the-better indicators:

$$\mu_{jk}^i = \frac{x_{jk}^i}{\left[x_{j(max)}^i + x_{j(min)}^i\right]} \qquad (2)$$

4.2.3.2 Pair-Wise Comparisons

After normalization of technical indicators by fuzzy sets, it is noticeable that, if the variation of an indicator for all cities $\{\mu_{jk}^i | k = 1 \cdots K, \forall j = 1 \cdots m_i\}$ is larger than that of the other indicator $\{\mu_{lk}^i | k = 1 \cdots K, \forall l \neq j\}$, criterion j is expected to be more influential than criterion l when evaluating city k. Such observation enables us to employ the standard deviation of indicators to determine which criterion is more important and to what extent. The calculation of standard deviation, s_j^i, is given by:

$$s_j^i = \sqrt{\textstyle\sum_{k=1}^{K} (\mu_{jk}^i - \bar{\mu}_j^i)^2 / (K-1)} \qquad j = 1 \ldots m_i \qquad (3)$$

Then, a pair-wise comparison matrix $A_i = (a_{jl}^i)_{m_i x m_i}$ is created to measure the relative importance of criterion j over criterion l, given by:

$$a_{jl}^{i} = \frac{s_j^i - s_l^i}{s_{max}^i - s_{min}^i} \times \left(a_{m_i} - 1\right) + 1, \qquad s_j^i \geq s_l^i \tag{4}$$

$$a_{jl}^{i} = \frac{1}{\left[\frac{s_l^i - s_j^i}{s_{max}^i - s_{min}^i} \times \left(a_{m_i} - 1\right) + 1\right]} \qquad s_j^i < s_l^i \tag{5}$$

where $a_{m_i} = min\{9, int\left(\frac{s_{max}^i}{s_{min}^i} + 0.5\right)\}$ is a comparison scale (range from 1 to 9) for all criteria.

4.2.3.3 Weight Determination

According to theory of AHP analysis, if a_{jl}^i can consistently or correctly reflect the importance of technical criterion j over criterion l, we will have $a_{jl}^i = w_j^i / w_l^i$. Then, the following three laws will hold: (a) $a_{jj}^i = w_j^i / w_j^i = 1$; (b) $a_{jl}^i = w_j^i / w_l^i = 1/a_{lj}^i$; and (c) $a_{jl}^i \cdot a_{lp}^i = (w_j^i / w_l^i) \cdot (w_l^i / w_p^i) = w_j^i / w_p^i = a_{jp}^i$. Therefore, one can obtain the weight for each criterion by solving the following linear equations:

$$\sum_{j=1}^{m_i} \sum_{l=1}^{m_i} \left| a_{jl}^i \cdot w_l^i - w_j^i \right| = 0 \qquad j, l = 1 \ldots m_i, i = 1 \cdots n \tag{6}$$

$$w_j^i > 0, j = 1 \ldots m_i, i = 1 \cdots n \tag{7}$$

$$\sum_{j=1}^{m_i} w_j^i = 1 \text{ , } i = 1 \cdots n \tag{8}$$

However, as mentioned in many previous studies (Bryson, 1995; Jin *et al.*, 2004; Saaty, 1980; Sudhakar and Shrestha, 2003; Yu, 2002), it is usually difficult in practice to obtain a completely consistent pair-wise comparison matrix that satisfies the aforementioned three laws. Thus, this study has proposed the following non-linear optimization model to estimate the weights $\{w_j^i | j = 1 \ldots m_i, i = 1 \cdots n\}$ from the inconsistent a_{jl}^i:

$$min\, CIC(m_i) = \sum_{j=1}^{m_i} \sum_{l=1}^{m_i} \frac{\left| y_{jl}^i - a_{jl}^i \right|}{{m_i}^2} + \sum_{j=1}^{m_i} \sum_{l=1}^{m_i} \frac{\left| y_{jl}^i w_l^i - w_j^i \right|}{{m_i}^2} \tag{9}$$

$$y_{jj}^i = 1, j = 1 \ldots m_i, i = 1 \cdots n \tag{10}$$

$\frac{1}{y^i_{lj}} = y^i_{jl} \in |a^i_{jl} - da^i_{jl}, a^i_{jl} - da^i_{jl}| (j = 1, \ldots, m_i, l = j + 1, \ldots, m_i, i = 1 \cdots n)$ (11)

$w^i_j > 0, j = 1 \ldots m_i, i = 1 \cdots n$ (12)

$\sum_{j=1}^{m_i} w^i_j = 1 , i = 1 \cdots n$ (13)

In the above equations, $Y_i = (y^i_{jl})_{m_i x m_i}$ isdefined as the consistency judgment matrix, which is adjusted based on $A_i = (a^i_{jl})_{m_i x m_i}$ during the minimizing process of the consistency index coefficient, denoted by $CIC(m_i)$. It consists of the following two parts:

- Minimization of $\sum_{j=1}^{m_i} \sum_{l=1}^{m_i} \frac{|y^i_{jl} - a^i_{jl}|}{m_i{}^2}$ to match the judgment matrix $\boldsymbol{Y_i} = (\boldsymbol{y^i_{jl}})_{\boldsymbol{m_i x m_i}}$ with the original comparison matrix $\boldsymbol{A_i} = (\boldsymbol{a^i_{jl}})_{\boldsymbol{m_i x m_i}}$ as closely as possible so that $\boldsymbol{Y_i} = (\boldsymbol{y^i_{jl}})_{\boldsymbol{m_i x m_i}}$ can reflect the original comparison information to the maximum extent; and
- Minimization of $\sum_{j=1}^{m_i} \sum_{l=1}^{m_i} \frac{|y^i_{jl} w^i_l - w^i_j|}{m_i{}^2}$, functions to ensure that $\boldsymbol{Y_i} = (\boldsymbol{y^i_{jl}})_{\boldsymbol{m_i x m_i}}$ be as consistent as possible to satisfy Eqs. (6)-(8).

Constraints (10) and (11) limit that all the elements in $A_i = (a^i_{jl})_{m_i x m_i}$ should satisfy the first two aforementioned laws. Note that the third law is not included in the constraints since it is considered by the second part of the objective function. In addition, constraint (11) introduces a non-negative parameter d to measure the deviation degree between $Y_i = (y^i_{jl})_{m_i x m_i}$ and $A_i = (a^i_{jl})_{m_i x m_i}$. Constraint (12) ensures the non-negative weights, and constraint (13) limits the sum of all weights equal to 1.

Solving the proposed optimization model yields two types of information: 1) the judgment matrix $Y_i = (y^i_{jl})_{m_i x m_i}$, and 2) the vector of weights for different technical criteria $\{w^i_j > 0, j = 1 \ldots m_i\}$. However, the global optimal solutions are not assured for the proposed optimization model due to its non-convexity attribute. Thus, this study has employed the convergence criterion of $CIC(m_i) \leq 0.1$ to ensure that the obtained judgment matrix $Y_i = (y_{jl}^i)_{m_i x m_i}$ is consistent based on extensive numerical experiments.

4.2.3.4 Weight Integration and Synthesis

This step synthesizes the weights obtained from the policy level and technical level to get the final evaluation score for each city, given by:

$$s_k = \sum_{i=1}^{n}\left[w_i \cdot \sum_{j=1}^{m_i}(\mu_{jk}^i \cdot w_j^i)\right] \tag{14}$$

The synthesis results will reflect the overall preference to the cities to be evaluated with respect to the goal.

4.2.4 Case Study

To illustrate the applicability of the proposed framework and models in evaluating the level of urban transit system development, this study has selected 9cities in the Chongqing metropolitan area, China for a case study. Chongqing is situated in the upper reaches of the Yangtze River at the confluence of the Yangtze and Jialing Rivers in southwest China. With an area of 82,400 square kilometers (31,800 square miles) and 30 million population, it is the biggest municipality (in terms of area and population size) under direct administration by the Chinese central government. In recent years, Chongqing has made great efforts to develop the public transport. By the end of 2014, more than 8000 buses were at service and 500 bus routes and 6 metro lines were in operation. Meanwhile, Chongqing has been selected as one of the Tier-1Transit Metropolis demonstration cities by the Ministry of Transport of China. In the past five years, transit network optimization and planning have been undertaken to boost public transport development and improve its performance. With data collected from 9 cities in the metropolitan area, this case study aims to apply the proposed framework and models to evaluate and compare their overall levels of transit system development as well as to reveal their microscopic performances in specific areas for improvement.

4.2.5.1 Data Collection

Data used in this study were collected from the Chongqing bus company and from surveys conducted in 2012.Specifically, detailed information on transit operation, facilities, safety and emission were collected to calculate relevant technical indicators. Survey was conducted to measure the load factors as well as the bus running speeds in critical sections. In-vehicle survey was also conducted to estimate the transfer rates and bus punctuality.

With the above input information, this study has computed the value of each technical indicator corresponding to different cities, as summarized in Table 4.2.

Table 4.2 The model input

Criteria		Cities								
Policy Level	Technical Level	Nan'an	Jiangbei	Yubei	Yuzhong	Beibei	Jiulongpo	Dadukou	Shapingba	Banan
C-1 Transit Development Priority	C-1-1 Mode share of transit	26%	25%	24%	27%	29%	26%	27%	27%	27%
	C-1-2 Operation subsidy	0.6	0.7	0.8	0.5	0.4	0.6	0.7	0.6	0.5
	C-1-3 Investment in public transport's fixed assets	55%	58%	54%	56%	55%	54%	54%	56%	54%
	C-1-4 Average income level of public transport drivers	105%	110%	120%	117%	109%	110%	109%	110%	110%
	C-1-5 Area of depots and stations per bus	77	85	87	73	81	71	86	75	87
	C-1-6 Bus priority lanes	23%	21%	22%	24%	27%	29%	22%	25%	21%
C-2 Transit Facility Construction	C-2-1 Coverage of transit stations	67%	71%	72%	69%	69%	66%	66%	75%	72%
	C-2-2 Density of public transport network	3.8	4.1	3.5	3.8	3.4	3.5	3.4	3.7	3.5
	C-2-3 Capacity of public transport	890	920	916	936	902	953	870	984	986
	C-2-4 Penetration of IC card usage	51%	56%	55%	53%	58%	50%	50%	57%	48%
C-3 Transit Operation and Service	C-3-1 Complaint rate	0.9	0.7	0.5	0.8	0.7	0.8	0.9	0.6	0.8
	C-3-2 Satisfaction ratio of public transport users	95%	98%	97%	96%	93%	95%	94%	96%	96%
	C-3-3 Load factor in the peak hour	85%	83%	78%	88%	81%	85%	83%	88%	83%
	C-3-4 Average transfer rate	1.5	1.6	1.3	1.6	1.2	1.4	1.3	1.5	1.2
	C-3-5 Punctuality	75%	78%	77%	73%	77%	76%	76%	75%	78%
	C-3-6 Average bus speed	17.5	16.1	20.2	17.9	19.2	18.8	20.2	16.9	22.6
C-4 Transit Safety	C-4-1 Fatality rate	0.07	0.09	0.06	0.09	0.04	0.06	0.08	0.09	0.05
	C-4-2 Accident rate	0.8	0.9	0.7	0.6	0.8	0.6	0.7	0.9	0.6
C-5 Sustainability	C-5-1 Energy consumption	290	281	284	292	287	282	297	298	285
	C-5-2 CO2 emission	65	60	65	68	68	68	67	61	59
	C-5-3 Percentage of green buses	85%	80%	88%	81%	82%	83%	85%	80%	82%

4.2.5.2 The Evaluation Process and Results

A group of unbiased professionals were nominated to participate in the weighting process for policy preferences. Participants were asked to assign a weight to each policy criterion according to a scale of numbers indicating how many times more important or dominant one element is over another. The numbers from 1 to 9 is quantify the importance degrees (e.g. 1 means equal importance, 3 means moderate importance, and 9 is for extreme importance). Meanwhile, if criterion *i* has one of the above non-zero numbers assigned to it when compared with criterion *j*, then *j* has the reciprocal with *i* value when compared.

A set of five weights for five policy criteria was calculated by the eigenvalue method, given by$\{w_i|i = 1, \ldots ,5\} = \{0.517,0.132,0.198,0.107,0.046\}$. One can observe that the criteria of transit development priority (C-1) and transit operation and service (C-3) are deemed to be relatively more important by the experts and decision makers in the Chongqing metropolitan area.

According to their definitions, the technical criteria of “complaint rate”, “load factor in the peak hour”, “average transfer rate”, “fatality rate”, “accident rate”, “energy consumption” and “CO2 emission” are considered as “the-lower-the-better” indicators, which will be processed with Eq. (1); while the remaining indices are taken as “the-higher-the-better” ones and are computed by Eq. (2). Further, the deviation of each technical criterion was calculated by Eq. (3). All fuzzy values and the standard deviations of technical indicators are calculated and listed in Table 4.3.

Table 4.3 Results of fuzzy scaling and normalization

Criteria		Cities									Standard Deviation
Policy Level	Technical Level	Nan'an	Jiangbei	Yubei	Yuzhong	Beibei	Jiulongpo	Dadukou	Shapingba	Banan	
C-1 Transit Development Priority	C-1-1 Mode share of transit	0.491	0.472	0.453	0.509	0.547	0.491	0.509	0.509	0.509	0.027
	C-1-2 Operation subsidy	0.500	0.583	0.667	0.417	0.333	0.500	0.583	0.500	0.417	0.102
	C-1-3 Investment in public transport's fixed assets	0.491	0.518	0.482	0.500	0.491	0.482	0.482	0.500	0.482	0.012
	C-1-4 Average income level of public transport drivers	0.467	0.489	0.533	0.520	0.484	0.489	0.484	0.489	0.489	0.020
	C-1-5 Area of depots and stations per bus	0.487	0.538	0.551	0.462	0.513	0.449	0.544	0.475	0.551	0.040
	C-1-6 Bus priority lanes	0.460	0.420	0.440	0.480	0.540	0.580	0.440	0.500	0.420	0.055
C-2 Transit Facility Construction	C-2-1 Coverage of transit stations	0.475	0.504	0.511	0.489	0.489	0.468	0.468	0.532	0.511	0.022
	C-2-2 Density of public transport network	0.507	0.547	0.467	0.507	0.453	0.467	0.453	0.493	0.467	0.031
	C-2-3 Capacity of public transport	0.480	0.496	0.494	0.504	0.486	0.513	0.469	0.530	0.531	0.022
	C-2-4 Penetration of IC card usage	0.472	0.519	0.509	0.491	0.537	0.463	0.463	0.528	0.444	0.033
C-3 Transit Operation and Service	C-3-1 Complaint rate	0.357	0.500	0.643	0.429	0.500	0.429	0.357	0.571	0.429	0.095
	C-3-2 Satisfaction ratio of public transport users	0.497	0.513	0.508	0.503	0.487	0.497	0.492	0.503	0.503	0.008
	C-3-3 Load factor in the peak hour	0.488	0.500	0.530	0.470	0.512	0.488	0.500	0.470	0.500	0.019
	C-3-4 Average transfer rate	0.464	0.429	0.536	0.429	0.571	0.500	0.536	0.464	0.571	0.056
	C-3-5 Punctuality	0.497	0.517	0.510	0.483	0.510	0.503	0.503	0.497	0.517	0.011
	C-3-6 Average bus speed	0.452	0.416	0.522	0.463	0.496	0.486	0.522	0.437	0.584	0.052
C-4 Transit Safety	C-4-1 Fatality rate	0.462	0.308	0.538	0.308	0.692	0.538	0.385	0.308	0.615	0.144
	C-4-2 Accident rate	0.467	0.400	0.533	0.600	0.467	0.600	0.533	0.400	0.600	0.082
C-5 Sustainability	C-5-1 Energy consumption	0.499	0.515	0.509	0.496	0.504	0.513	0.487	0.485	0.508	0.011
	C-5-2 CO_2 emission	0.488	0.528	0.488	0.465	0.465	0.465	0.472	0.520	0.535	0.029
	C-5-3 Percentage of green buses	0.506	0.476	0.524	0.482	0.488	0.494	0.506	0.476	0.488	0.016

After normalization of all the technical with the fuzzy sets, five pair-wise comparison matrices corresponding to five policy criteria $A_i = (a_{jl}^i)_{m_i x m_i}, i = 1\ to\ 5$ can be constructed with Eq. (4) and Eq. (5), each measuring the relative importance of technical criterion j over technical criterion l under the policy criterion i.The non-linear optimization model is then run for each comparison matrix to maximize its judgment consistency and estimate the weights for various technical criteria (see Table 4.4). Weights from the policy level and the technical level are finally synthesized with Eq. (14) to obtain the ranking scores of each city with respect to different policy and technical criteria (see Table 4.5).

Table 4.4 Ranking scores with respect to different technical criteria

Criteria					Cities								
Policy	Policy Criteria Weight	Technical	Technical Criteria Weight	CIC	Nan'an	Jiangbei	Yubei	Yuzhong	Beibei	Jiulongpo	Dadukou	Shapingba	Banan
C-1	0.518	C-1-1	0.104	0.0251	0.026	0.025	0.024	0.027	0.03	0.026	0.027	0.027	0.027
		C-1-2	0.552		0.143	0.166	0.19	0.119	0.095	0.143	0.166	0.143	0.119
		C-1-3	0.066		0.017	0.018	0.017	0.017	0.017	0.017	0.017	0.017	0.017
		C-1-4	0.072		0.017	0.018	0.02	0.019	0.018	0.018	0.018	0.018	0.018
		C-1-5	0.091		0.023	0.025	0.026	0.022	0.024	0.021	0.026	0.022	0.026
		C-1-6	0.115		0.027	0.025	0.026	0.028	0.032	0.034	0.026	0.03	0.025
C-2	0.132	C-2-1	0.178	0.0004	0.011	0.012	0.012	0.011	0.011	0.011	0.011	0.012	0.012
		C-2-2	0.315		0.021	0.023	0.019	0.021	0.019	0.019	0.019	0.02	0.019
		C-2-3	0.168		0.011	0.011	0.011	0.011	0.011	0.011	0.01	0.012	0.012
		C-2-4	0.339		0.021	0.023	0.023	0.022	0.024	0.021	0.021	0.024	0.02
C-3	0.199	C-3-1	0.574	0.0409	0.041	0.057	0.073	0.049	0.057	0.049	0.041	0.065	0.049
		C-3-2	0.061		0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
		C-3-3	0.069		0.007	0.007	0.007	0.006	0.007	0.007	0.007	0.006	0.007
		C-3-4	0.121		0.011	0.01	0.013	0.01	0.014	0.012	0.013	0.011	0.014
		C-3-5	0.063		0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
		C-3-6	0.111		0.01	0.009	0.011	0.01	0.011	0.011	0.011	0.01	0.013
C-4	0.107	C-4-1	0.675	0	0.033	0.022	0.039	0.022	0.05	0.039	0.028	0.022	0.045
		C-4-2	0.325		0.016	0.014	0.019	0.021	0.016	0.021	0.019	0.014	0.021
C-5	0.046	C-5-1	0.12	0	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
		C-5-2	0.691		0.015	0.017	0.015	0.015	0.015	0.015	0.015	0.016	0.017
		C-5-3	0.189		0.004	0.004	0.005	0.004	0.004	0.004	0.004	0.004	0.004

Table 4.5 Ranking scores with respect to different policy criteria

Cities	Criteria					The overall score	Ranking
	C-1 Transit Development Priority	C-2 Transit Facility Construction	C-3 Transit Operation and Service	C-4 Transit Safety	C-5 Sustainability		
Nan'an	0.254	0.064	0.081	0.050	0.023	0.470	7
Jiangbei	0.278	0.069	0.096	0.036	0.024	0.502	2
Yubei	0.303	0.065	0.118	0.058	0.023	0.566	1
Yuzhong	0.233	0.066	0.088	0.043	0.022	0.451	9
Beibei	0.216	0.065	0.101	0.066	0.022	0.470	8
Jiulongpo	0.259	0.062	0.091	0.060	0.022	0.494	3
Dadukou	0.280	0.061	0.084	0.046	0.022	0.494	4
Shapingba	0.257	0.068	0.105	0.036	0.023	0.490	5
Banan	0.232	0.063	0.095	0.065	0.024	0.479	6

4.2.5.3 Discussions

From the above analyses, one can observe that the Yubei city has the top overall ranking within the nine cities, indicating its best performance in transit system development. In addition to yielding the overall ranking for all cities, the proposed evaluation framework and model can also generate ranking scores of a city with respect to a specific criterion. As shown in Table 5, Yubei's higher overall ranking is attributed to its better performance in criteria C-1 (Transit Development Priority, 0.303) and C-3(Transit Operation and Service, 0.118); however, Yubei's relatively lows core (0.065) in criterion C-2 indicates its weakness in transit facility construction. Despite their relatively low overall raking, Beibei and Banan rank the first in criteria "Transit Safety" and "Sustainability", respectively. Jiulongpo and Dadukou rank the third and fourth behind Yubei and Jiangbei with a balanced performance in all aspects. Regarding the worst case, Yuzhong city, it has a relatively poor performance in both Transit Development Priority and Transit Operation and Service which would be a target for further improvement.

Deficiencies of cities with respect to a specific criterion can also be easily identified by comparing their ranking scores under that criterion. For examples, Beibei needs to make more efforts in the transit development priority, Nan'an needs to improve their transit operation services, while Jiangbei and Yuzhong city should pay more attention to transit safety and sustainability, respectively. Jiulongpo and Beibei have a better performance in the indicator of Bus Priority Lane while Jiangbei and Banan need to contribute more resources in increasing the mileage of their bus lanes. When looking at the "Complaint Rate", an important indicator to transit services, Yubei holds a safe lead position, while Dadukou and Nanan should improve their transit service quality to decease the complaint rate. There are no significant differences in ranking scores across different cities when evaluating the satisfaction ratio of public transport users, load factor in the peak hour, and punctuality, reflecting similar development levels of those cities under those criteria. The above information is valuable for transportation authorities to identify deficiencies and areas of improvement for a city in comparison with other peer cities.

4.2.5 Conclusion

This section presents a multi-dimensional evaluation framework at city/regional level which contains the policy level and the technical level to compare the performance of different cities/regions in the development of public transport system. The "policy level" is designed to capture a city's characteristics and developing priorities as well as the subjective opinions of various transit stakeholders during the evaluation process, while the "technical level" functions to compare and assess detailed technical indicators with an enhanced multi-criteria ranking model. The proposed framework offers the advantage of preventing the vagueness and uncertainty of the decision-maker(s) when evaluating technical criteria while properly retaining the policy preferences from decision makers. In this study, a total of 21 technical criteria are classified into five policy categories for evaluation of the transit system development. Note that the proposed evaluation framework and model offers the flexibility to include or exclude criteria during evaluation.

It selects nine cities in the Chongqing metropolitan area for a case study. Results reveal that the proposed evaluation framework and model can effectively generate the overall rankings of different cities/regions in transit system development and also identify microscopic deficiencies and areas of improvement for a city with respect to any specific criterion.

4.3. Operator-level Transit System Efficiency Assessment

4.3.1 Research Motivation

Unlike city or regional level transit system, the bus company, as a business operation-orientated unit, is always required to measure its productivity by testing the relationship between allocated resources and corresponding outputs. It directly leads that many researchers have assumed transit system as production lines, and evaluated the performance of such lines by comparing multiple inputs and outputs when measuring bus operator's efficiency (Fare and Grosskopf, 1996, 2000; Seiford and Zhu, 1999; Boile, 2001; Nolan *et al.* 2002; Sexton and Lewis, 2003; Zhu, 2003; Karlaftis, 2004; Nakanishi and Falcocchi, 2004; Hwang and Kao, 2006; Tsamboulas, 2006; Barnum et al., 2008; Kao and Hwang, 2008; Sheth *et al.*, 2007; Lao and Liu, 2009; Sanchez, 2009; Yu and Fan, 2009; Zhao et al., 2011; Hawas et al., 2012; Karlaftis and Tsamboulas, 2012). Thus, most of these researchers used the Data Envelopment Analysis (DEA), a non-parametric method introduced by Farrell (1957) and popularized by Charnes *et al.* (1978). It is a managerial approach to assess relative performance/efficiency for evaluating decision making units (DMUs). Each DMU selects its best set of weights corresponding to consider inputs and outputs; the values of weights may thus vary from one DMU to another. The DEA models then calculate each DMU's performance score ranging between zero and one that represents its relative degree of efficiency (Wei and Chang, 2011).

As ever-increasing applications of DEA are applied in the transit efficiency assessment, some critical issues are deserved further investigation. Halme M *et al*. (1999) has pointed out that DEA calculations are traditionally value-free and the underlying assumption is that no output or input is more important than the others, although in the real-world there generally exists a Decision Maker (DM) who has preferences over outputs and inputs. Nevertheless, the different importance over different input or output indicators is an obvious case one cannot ignore when the systems' efficiency are reviewed. Andersen and Christian (1993) stated that DEA evaluates the relative efficiency of decision-making units but does not allow for ranking of the efficient units themselves. Both of the issues are constraints to widely and extensively apply DEA in system efficiency assessment.

To remedy such limitations, some efforts of combining the analytic hierarchy process (AHP) with the DEA have been made to complement each other. Bowen (1990) suggested a two-step process in site selection, where the first step is to apply the DEA to exclude numerically inefficient sites and the second step is to apply the AHP for further ranking the DEA-efficient sites. A similar method was also applied to manage investments in the various parts (sub-systems) of the State Economic Information System of China by Zhang and Cui (1999). Comparing it with the above methods, Shang and Sueyoshi (1995) proposed a reversal process to select the most appropriate and flexible alternative, which firstly uses AHP to quantify all the alternatives and then uses DEA to determine the most suitable one. Additionally, Sinuany-Stern *et al.* (2000) presented an interesting AHP/DEA methodology for fully ranking organizational units with multiple inputs and multiple outputs. They suggested running DEA for each pair of indicators separately and further choosing efficiency number to generate the pair-wise matrix, which could be used by AHP model in the steps ahead. A hierarchical AHP/DEA methodology for the facilities layout design was proposed by Yang and Kuoin (2003) and Ertay *et al.* (2006). Moreever, Ramanathan (2006) developed a DEAHP model, which uses DEA to generate local weights of alternatives from pair-wise comparison matrices and AHP to aggregate the local weights of alternatives over all the criteria.

Despite many constructive efforts in combing AHP and DEA, most existing studies used AHP and DEA separately rather than inherently integrating them into a unified model. As a result, the objective of this section is to develop an enhanced DEA model with sufficient flexibility to capture the inherent preference information over input and output indicators, and further apply the proposed model to evaluate the efficiency transit operators. This part will focus on the following critical research tasks:

- Proposes a robust enhanced DEA model to effectively take into account the preferences information over indicators, which features the integration of a Fuzzy-AHP model introduced in **Section 4.2** to generate cone constraints for the conventional DEA;
- Offers the advantage in breaking the tie between those efficient units under the conventional DEA;
- Apply the proposed model into a real world case to demonstrate the model's applicability.

4.3.2 The Proposed Model

4.3.2.1 Notation

To facilitate the model presentation, all definitions and notations used hereafter are summarized in Table 4.6.

Table 4.6 Notation of key parameters used in the proposed model

I	Index corresponding to indicator in input group$(i = 1 \ldots m)$;
K	Index corresponding to indicator in output group$(k = 1 \ldots s)$;
J	Index corresponding to DMU$(j = 0 \ldots n)$;
v_{i}	The weight of input indicator $(i = 1 \ldots m)$;
W_k	The weight of output indicator $(i = 1 \ldots s)$;
p_j	The efficiency of DMU *j*(j=0…n);
x_{ij}	The value of input indicator *i* corresponding to DMU *j*
y_{kj}	The value of output indicator *k* corresponding to DMU *j*
μ_{ij}	Fuzzy membership value corresponding tox_{ij}
$\bar{\mu}_i$	Average fuzzy membership value for indicator *i*
$x_{i(min)}$	The minimal crisp value for input indicator *i*
$x_{i(mid)}$	The medium crisp value for input indicator *i*
$x_{imax)}$	The maximal crisp value for input indicator *i*
s_i	Standard deviation of indicator values corresponding to input indicator *i*
s_{min}	*min* $\{s_i \mid i = 1,. . .,n\}$
s_{max}	*max* $\{s_i \mid i = 1,. . .,n\}$
$A = (a_{ij})_{nxn}$	Pair-wise comparison matrix
$\mathrm{a_m}$	Comparison scale for the pair-wise comparison matrix
w_i	Weight for criterion *j*
$Y = (y_{ij})_{nxn}$	Consistency judgment matrix
C.I.C.(n)	Consistency index coefficient
$(y_{input})_{mxm}$	The input group pair-wise matrix
$(y_{output})_{sxs}$	The output group pair-wise matrix
λ_{input}	The max eigenvalue of input pair-wise matrix
$\lambda_{\mathrm{output}}$	The max eigenvalue of output pair-wise matrix

4.3.3.2 Selection of Input and Output Indicators

The proposed model is based on the concept of evaluating performance according to some selected criteria. Thus, a set of representative indicators associated with transit operator performance is recommended to select data for the proposed model. In accordance with the theory of DEA models, the targeted indicators are classified into two groups: the input group and the output group. The input group includes the indicators that are relevant to allocate passenger service resources, for example, cost structure, bus fleet, human resources, etc. Meanwhile, the output indicators are intended to reflect resource allocation based on goals such as passenger volume, operating mileage and customer satisfaction. Normally, the selected indicators are widely available, easily collected, and customized to fit the local situation.

4.3.2.3 Introduction of Constraint Cones into DEA

Wu *et al.*, (1999) firstly introduced a concept of AHP restraint cone to be utilized by conventional DEA model. The model functions in keeping characteristics of the conventional DEA model as well as capturing preferences of the decision makers by adding the constraint cones.

Along the line of Wu's work, two constraint cones, $(y_{input})_{m\times m}$ and $(y_{output})_{s\times s}$, containing weights are defined for input and output group respectively. Both of them are later embedded into conventional DEA model, given by:

$$Max\ p_0 = (v^T Y_{k0}) \quad (1)$$

$$s.t. W^T X_{ij} - v^T Y_{kj} \geq 0 \quad j = 1,2,3, \ldots, J; i = 1, \ldots, N; k = 1, \ldots, M; \quad (2)$$

$$W^T X_{i0} = 1 \quad (3)$$

$$W^T * \{(y_{input})_{m\times m} - \lambda_{input} * E_n\} \geq 0, W \geq 0 \quad (4)$$

$$v^T * \{ (y_{output})_{s\times s} - \lambda_{output} * E_s\} \geq 0, v \geq 0 \quad (5)$$

However, the main limitation of *Wu's* enhanced DEA model is to employ the conventional AHP model to generate constraint cones, where some critical issues deserved further investigation, specifically, 1) how to handle the very unbalanced scale of judgment, 2) how to properly construct the pair-wise comparison matrix subject to the biased impacts from the objective judgment, selection and preference of decision-makers.

To resolve this diagnostic problem, this section re-employed the Fuzzy-AHP model introduced in **Section 4.2** to generate constraint cones. It features the integration of the fuzzy logic with a hierarchical AHP structure to: 1) normalize the scales of different evaluation indicators, 2) construct the matrix of pair-wise comparisons with the fuzzy set, and 3) optimize the weight of each criterion with a non-linear programming model to maximize consistency.

Figure 4.3 below illustrates the logical relationship between DEA and Fuzzy-AHP.

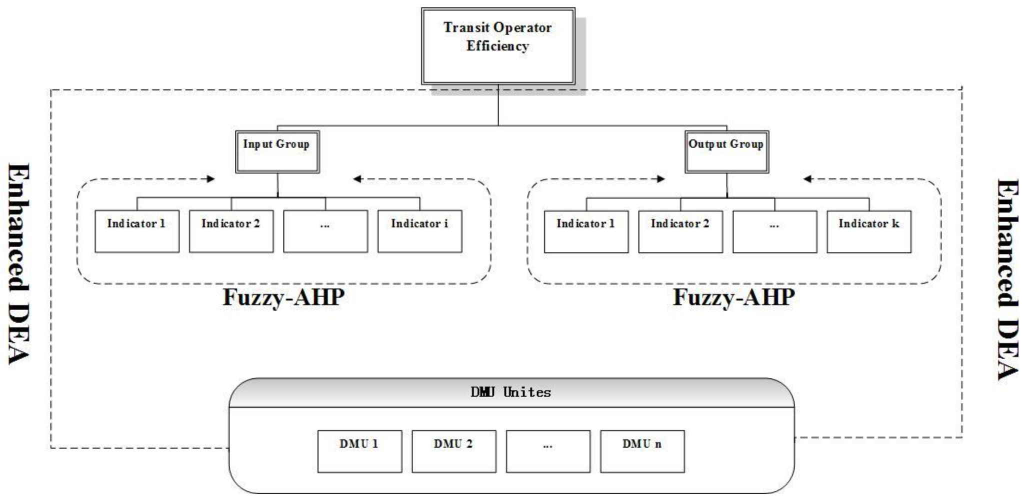


Figure 4.3 The proposed model structure

4.3.2.4 Construction of the Fuzzy-AHP Constraint Cones

Step 1: Fuzzy scaling

Considering the difficulty in comparing various criteria with different units, this step have employed a set of fuzzy membership functions to normalize the scales of different indicators, based on the characteristics of selected criterion. Two types of indicators, i.e. "the-lower-the-better" and "the-higher-the-better" are identified to normalize x_{ik} with their fuzzy sets, given by:

For the-lower-the-better indicators:

$$\mu_{ik} = \frac{[x_{i(max)+}x_{i(min)}-x_{ik}]}{[x_{i(max)+}x_{i(min)}]} \tag{6}$$

For the-higher-the-better indicators:

$$\mu_{ik} = \frac{x_{ik}}{[x_{i(max)+}x_{i(min)}]} \tag{7}$$

Step 2: Pair-wise comparison

After the normalization of all the indicators by fuzzy scaling, it is noticeable that, if the variation of an indicator for all operators $\{\mu_{ik}|k = 1 \cdots m, i = 1 \cdots n\}$ is larger than that of the other indicator$\{\mu_{jk}|k = 1 \cdots m, j \neq i\}$, criterion *i* is expected to be more influential than criterion *j* when evaluating operator *k*. The calculation of standard deviation s_i is given by the following equation:

$$S_i = \sqrt{\sum_{k=1}^{m}(\mu_{ik} - \bar{\mu}_i)^2/(m-1)} \tag{8}$$

Then, a pair-wise comparison matrix $A = (a_{ij})_{nxn}$ is calculated by Eq. (13) and (14) to measure the relative importance of criterion *i* over criterion *j*.

$$a_{ij} = \frac{S_i - S_j}{S_{max} - S_{min}} \times (a_m - 1) + 1, \qquad S_i \geq S_j \tag{9}$$

$$a_{ij} = \frac{1}{\left[\frac{S_j - S_i}{S_{max} - S_{min}} \times (a_m - 1) + 1\right]} \qquad S_i < S_j \tag{10}$$

Where $a_m = min\{9, int\left(\frac{S_{max}}{S_{min}} + 0.5\right)\}$ is a comparison scale for all criteria recommended by Jin *et al.* (2004).

Step 3: Consistency maximization

According to theory of AHP analysis, if a_{ij} can consistently or correctly reflect the importance of technical criterion *i* over criterion *j*, we will have $a_{ij} = w_i/w_j$. Then, the following three laws will hold: (a)$a_{ii} = w_i/w_i = 1$; (b)$a_{ij} = w_i/w_j = 1/a_{ji}$; and (c)$a_{ij} \cdot a_{jk} = (w_i/w_j) \cdot (w_j/w_k) = w_i/w_k = a_{ik}$. Therefore, one can obtain the weight for each criterion by solving the following linear equations:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \left|a_{ij} \cdot w_j - w_i\right| = 0 \tag{11}$$

$$w_i > 0, i = 1 \dots n \tag{12}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{13}$$

However, as mentioned in many previous studies (Bryson, 1995; Jin *et al.*, 2004; Saaty, 1980; Sudhakar and Shrestha, 2003; Yu, 2002), it is usually difficult in practice to obtain a completely consistent pair-wise comparison matrix that satisfies the aforementioned three laws. Thus, this study has proposed the following non-linear optimization model to estimate the weights $\{w_i | i = 1 \cdots n\}$ from the inconsistent a_{ij}:

$$\boldsymbol{min\, CIC(n)} = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{|y_{ij} - a_{ij}|}{n^2} + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{|y_{ij} w_j - w_i|}{n^2} \tag{14}$$

$$y_{ii} = 1 (i = 1, \dots, n) \tag{15}$$

$$\frac{1}{y_{ij}} = y_{ij} \in \left|a_{ij} - da_{ij}, a_{ij} - da_{ij}\right| (i = 1, \dots, n; j = i + 1, \dots, n) \tag{16}$$

$$w_i > 0 \; (i = 1, \dots, n) \tag{17}$$

$\sum_{i=1}^{n} w_i = 1$ (18)

In the above equations, $Y = (y_{ij})_{nxn}$ is defined as the consistency judgment matrix which is adjusted based on $A = (a_{ij})_{nxn}$ during the minimization process of the consistency index coefficient, denoted by *C.I.C. (n)*. It consists of the following two parts:

- Minimization of $\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{|y_{ij}-a_{ij}|}{n^2}$ to match the judgment matrix $Y = (y_{ij})_{nxn}$ with the original comparison matrix $A = (a_{ij})_{nxn}$ as closely as possible so that $Y = (y_{ij})_{nxn}$ can reflect the original comparison information to the maximum extent; and
- Minimization of $\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{|y_{ij}-a_{ij}|}{n^2}$, functions to ensure that $Y = (y_{ij})_{nxn}$ be as consistent as possible to satisfy Eqs. (11) - (13).

Constraints (15) and (16) limit that all the elements in $A = (a_{ij})_{nxn}$ should satisfy the first two aforementioned laws. Note that the third law is not included in the constraints since it is considered by the second part of the objective function. In addition, constraint (16) introduces a non-negative parameter d to measure the deviation degree between $Y = (y_{ij})_{nxn}$ and $A = (a_{ij})_{nxn}$. Constraint (17) ensures the non-negative weights, and constraint (18) limits the sum of all weights equal to 1.

Solving the proposed optimization model yields two types of information: 1) the judgment matrix $\boldsymbol{Y} = (\boldsymbol{y_{ij}})_{\boldsymbol{nxn}}$, and 2) the vector of weights for different technical criteria $\{w_i > 0\ (i = 1, \ldots, n)\}$. However, the global optimal solutions are not assured for the proposed optimization model due to its non-convexity attribute. Thus, this study has employed the convergence criterion of $C.I.C.(n) \leq 0.1$ to ensure that the obtained judgment matrix $\boldsymbol{Y} = (\boldsymbol{y_{ij}})_{\boldsymbol{nxn}}$ is consistent based on extensive numerical experiments.

By processing the Fuzzy-AHP model for input and output group respectively, two optimized consistent pair-wise matrices, $(y_{input})_{m\times m}$ and$(y_{output})_{s\times s}$, are obtained to represent the constraint cones and ready to be utilized by conventional DEA.

4.3.2.5 Derivation

To prove formulation's validity and reliability, the derivation is given as following. Here, we take the constraint cone of input group as an example:

Definition 1:

The solution domains of $W^T[(y_{input})_{m\times m} - \lambda_{input}E_m] \geq 0$ *and* $W^T[(y_{input})_{m\times m} - \lambda_{output}E_m] = 0$ *are the same when the pair-wise matrix* $(y_{input})_{m\times m}$ *satisfies the consistency check of AHP requirement.*

It is required to calculate the maximum eigenvalue λ_{input} of matrix$(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}}$.

Set $\quad C = (\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} - \boldsymbol{\lambda_{input}E_m}$, where E_m is an m order unit matrix;

Since $\quad [(y_{input})_{m\times m} - E_m] \geq 0$ and $[(y_{input})_{m\times m} - \lambda_{input}E_m]W = CW \geq 0$;

Then $\quad [(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} - E_m][(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} - \lambda_{input}E_m]W \geq 0;$

$$[(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}}{}^2 - (\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} - \lambda_{input}(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} + \lambda_{input}E_m]W \geq 0; \text{ and}$$

$$\left\{\left[(y_{input})^2_{m\times m} - \lambda_{input}(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}}\right] - \left[(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} - \lambda_{input}E_m\right]\right\}W \geq 0;$$

Since $\quad (\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} = (y_{ij})_{m\times m}$ satisfies the consistency-check of AHP process;

Then $\quad y_{ij} = y_{ik}y_{kj}$ and $(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}}{}^2 = (y_{ij}) * (y_{ij}) = (\sum_{k=1}^{m} y_{ik}y_{kj}) = m(y_{ij})_{m*m}$;

$$[(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} - \boldsymbol{\lambda_{input}E_m}]W = CW \leq 0;$$

$$\left[(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} - E_m\right] W^T \left[(y_{input})_{m\times m} - \lambda_{input}E_m\right] = W^T[\lambda_{input}E_m - (\boldsymbol{y_{input}})_{\boldsymbol{m\times m}}]$$

$$= - W^T \left[(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} - \lambda_{input}E_m\right] \geq 0$$

Since $[(y_{input})_{m\times m} - E_m] \geq 0$ and $W^T[(y_{input})_{m\times m} - \lambda_{input}E_m] = CW \geq 0$;

Then $W^T\left[\lambda_{input}E_m - (y_{input})_{m\times m}\right] = 0.$

Definition 2:

The efficiency of the selected DMU obtained from enhanced DEA model is equal to the weighted average of the selected DMU obtained from AHP process, given by:

$$p_0{}^* = \frac{\sum_{k=1}^{s} v_k^* y_{k0}}{\sum_{k=1}^{m} w_k^* x_{k0}} * T$$

where *T* is a parameter; x_{k0} is the value of input indicator *k of DMU 0*, and y_{k0} is the value of output indicator *k of DMU 0.*

According to definition 1, $[(\boldsymbol{y_{input}})_{\boldsymbol{m\times m}} - \boldsymbol{\lambda_{input}E_m}]W = CW = 0$

Then, we have $W = KW^* \quad v = \overline{K}v^*$. The enhanced DEA model could be rewritten as:

$$Max\ p_0 = \overline{K}{v^*}^T Y_0 \tag{23}$$

$$s.t. K{w^*}^T X_0 - \overline{K}{v^*}^T Y_0 \geq 0, j = 1,2,\dots,n; \tag{24}$$

$$K{w^*}^T X_0 = 1; \tag{25}$$

Then, the max value is equal to $p_0{}^* = \overline{K}{v^*}^T Y_0$, where $\overline{K} = K * max_1^n \frac{{w^*}^T x_j}{{v^*}^T y_j}$ $and\ K = \frac{1}{{w^*}^T x_0}$;

so $p_0{}^* = \frac{{v^*}^T Y_0}{{w^*}^T X_0} * max_1^n \frac{{w^*}^T x_j}{{v^*}^T y_j}$ where $T = max_1^n \frac{{w^*}^T x_j}{{v^*}^T y_j}$;

4.3.3 Case Study

A real world case study of Nanjing City, the capital of Jiangsu province, is selected to illustrate the applicability of the proposed approach. The area of municipal district of Nanjing City is 6,598 square kilometers with over 7.4 million permanent residents. This study assesses efficiencies of seven bus companies severing Nanjing based on 2009 and 2010 datasets. Moreover, a comparison between conventional DEA and the proposed model is also performed.

Due to availability of original dataset access, fuel cost, labor cost, depreciation expenses and other costs have been collected as input indicators while the passenger volume, operated mileage and passenger service satisfaction have been chosen to be output indicators in this study. Table 4.7 and Table 4.8 record raw data used for 2009 and 2010 respectively.

Table 4.7 Data used for evaluation (Year 2009)

Year 2009	Fuel Cost (Yuan)	Labor Cost (Yuan)	Depreciation expense (Yuan)	Others (Yuan)	Patronage Volume (Trips)	Mileages (Km)	Satisfaction Index
Nanjing Bus	27728.101	44930.800	12484.722	2437.661	51428.510	17979.921	59.716
ZhongBei Bus	10712.022	14625.681	4218.512	1470.032	21505.301	7363.795	62.790
YaGao Bus	4778.875	4942.479	1813.263	757.199	7914.638	2823.705	53.588
XinCheng Bus	6116.101	8402.902	2230.166	600.331	10086.515	4896.807	50.794
XinNingPu Bus	2487.872	2355.404	653.507	565.992	4082.552	1600.342	56.675
PuKou Bus	1621.567	2541.051	515.642	209.532	2820.611	1618.651	60.492

LiuHe Bus	2898.059	3454.670	587.771	244.863	2856.341	2831.942	62.292

Table 4.8 Data used for evaluation (Year 2010)

Year 2010	Fuel Cost (Yuan)	Labor Cost (Yuan)	Depreciation expense (Yuan)	Others (Yuan)	Patronage Volume (Trip)	Mileages (Km)	Satisfaction Index
Nanjing Bus	32674.731	53715.450	13470.471	3368.88	50057.131	18581.940	60.870
ZhongBei Bus	12614.802	17051.800	4792.801	1293.802	20852.12	7381.976	63.770
YaGao Bus	5684.551	5897.242	2360.456	850.487	7591.782	2939.931	55.560
XinCheng Bus	7037.315	9621.091	2471.733	1399.178	8511.252	4754.776	50.480
XinNingPu Bus	2786.802	3058.442	822.224	560.712	4364.703	1755.605	56.680
PuKou Bus	2246.119	3119.237	620.478	288.791	3127.535	1878.909	61.830
LiuHe Bus	3706.318	4265.043	760.267	273.742	2947.177	2938.805	61.930

4.3.4.1 Construction of Constraint Cones

Step 1: Fuzzy scaling

This step has employed a set of fuzzy membership functions to normalize the scales of different indicators, based on the characteristics of each criterion. According to the definitions, all the input indicators here are considered as “the-lower-the-better”, which will be processed with Eq. (6) while the output indicators are taken as the-higher-the-better ones, and thus computed by Eq. (7). Further, the deviation of each technical criteria was calculated by Eq. (8). All of the fuzzy values and the standard deviations for 2009 and 2010, denoted as $\{\mu_{ij} | i = 1 \ldots 7, j = 1 \ldots 7\}$ and $\{s_j | j = 1 \ldots 7\}$, are listed in Tables 4.9 and 4.10.

Table 4.9 Fuzzy scaling for 2009 data

Year 2009	Nanjing Bus	ZhongBei Bus	YaGao Bus	XinCheng Bus	XinNingPu Bus	PuKou Bus	LiuHe Bus	s_j
Fuel Cost	0.055	0.635	0.837	0.792	0.915	0.945	0.901	0.313
Labor Cost	0.050	0.691	0.895	0.822	0.950	0.946	0.927	0.324
Depreciation Expense	0.040	0.676	0.861	0.828	0.950	0.960	0.955	0.330
Other Expenses	0.079	0.445	0.714	0.773	0.786	0.921	0.908	0.301
Patronage Volume	0.948	0.396	0.146	0.186	0.075	0.052	0.053	0.324
Mileages	0.918	0.376	0.144	0.250	0.082	0.083	0.145	0.298
Satisfaction Index	0.526	0.553	0.472	0.447	0.499	0.533	0.548	0.040

Table 4.10 Fuzzy scaling for 2010 data

Year 2010	Nanjing Bus	ZhongBei Bus	YaGao Bus	XinCheng Bus	XinNingPu Bus	PuKou Bus	LiuHe Bus	s_j
Fuel Cost	0.064	0.639	0.837	0.799	0.920	0.936	0.894	0.309
Labor Cost	0.054	0.700	0.896	0.831	0.946	0.945	0.925	0.322
Depreciation Expense	0.044	0.660	0.833	0.825	0.942	0.956	0.946	0.326
Other Expenses	0.075	0.645	0.767	0.616	0.846	0.921	0.925	0.296
Patronage Volume	0.944	0.393	0.143	0.161	0.082	0.059	0.056	0.322
Mileages	0.914	0.363	0.145	0.234	0.086	0.092	0.145	0.294
Satisfaction Index	0.533	0.558	0.486	0.442	0.496	0.541	0.542	0.041

Step 2: Pair-wise comparison

After normalization of all the indicators with the fuzzy sets, the pair-wise comparison matrices corresponding to the input and output groups are constructed respectively with Eq. (9) and Eq. (10), each measuring the relative importance of indicator *j* over indicator *i*.

The pair-wise matrix of “Fuel cost”, “Labor cost”, “Depreciation expense” and “ Other cost” in 2009 input group:

$$A_{input} = \begin{bmatrix} 1.000 & 0.759 & 0.710 & 1.332 \\ 1.318 & 1.000 & 0.916 & 1.649 \\ 1.409 & 1.091 & 1.000 & 1.741 \\ 0.751 & 0.606 & 0.575 & 1.000 \end{bmatrix}$$

The pair-wise matrix of “Passenger Volume”, “Mileage” and “Satisfaction Index” in 2009 output group:

$$A_{output} = \begin{bmatrix} 1.000 & 1.688 & 7.919 \\ 0.592 & 1.000 & 7.231 \\ 0.126 & 0.138 & 1.000 \end{bmatrix}$$

The pair-wise matrix of “Fuel cost”, “Labor cost”, “Depreciation expense” and “ Other cost” in 2010 input group:

$$A_{input} = \begin{bmatrix} 1.000 & 0.661 & 0.713 & 1.354 \\ 1.513 & 1.000 & 0.918 & 1.652 \\ 1.403 & 1.091 & 1.000 & 1.763 \\ 0.739 & 0.605 & 0.567 & 1.000 \end{bmatrix}$$

The pair-wise matrix of “the volume of Patronage”, “Mileage” and “Satisfaction Index” in 2010 output group:

$$A_{output} = \begin{bmatrix} 1.000 & 1.606 & 7.223 \\ 0.623 & 1.000 & 7.005 \\ 0.138 & 0.143 & 1.000 \end{bmatrix}$$

Step 3: Consistency maximization

After the construction of two original pair-wise matrices, the non-linear optimization model, Eq. (14)-Eq. (18), is then solved for each comparison matrix to maximize its judgment consistency. Eventually, two optimized pair-wise matrices corresponding to the input and output groups are obtained as the constraint cones.

The optimized pair-wise matrix of 2009 input indicator group (the input indicator group constraint cone) is given as following:

$$y_{input} = \begin{bmatrix} 1.000 & 0.789 & 0.738 & 1.385 \\ 1.267 & 1.000 & 0.953 & 1.715 \\ 1.355 & 1.049 & 1.000 & 1.810 \\ 0.722 & 0.583 & 0.552 & 1.000 \end{bmatrix}$$

The optimized pair-wise matrix of 2009 output indicator group (the output indicator group constraint cone) is given as following:

$$y_{output} = \begin{bmatrix} 1.000 & 1.755 & 8.236 \\ 0.570 & 1.000 & 7.521 \\ 0.121 & 0.133 & 1.000 \end{bmatrix}$$

The optimized pair-wise matrix of 2010 input indicator group (the input indicator group constraint cone) is given as following:

$$y_{input} = \begin{bmatrix} 1.000 & 0.664 & 0.713 & 1.354 \\ 1.507 & 1.000 & 0.918 & 1.652 \\ 1.403 & 1.089 & 1.000 & 1.763 \\ 0.739 & 0.605 & 0.567 & 1.000 \end{bmatrix}$$

The optimized pair-wise matrix of 2010 output indicator group (the output indicator group constraint cone) is given as following:

$$y_{output} = \begin{bmatrix} 1.000 & 1.606 & 7.223 \\ 0.623 & 1.000 & 7.005 \\ 0.138 & 0.143 & 1.000 \end{bmatrix}$$

The weights of indicators for year 2009 and year 2010 are summarized in Table 4.11.

Table 4.11 Weights of indicators in 2009 and 2010

Input Indicators	Weights in 2009	Weights (2010)
Fuel Cost	0.234	0.223
Labor Cost	0.277	0.289
Depreciation Expense	0.308	0.312
Other Expenses	0.181	0.175

Output Indicators	Weights (2009)	Weights (2010)
Passenger Volume	0.575	0.474
Mileages	0.352	0.460
Satisfaction Index	0.073	0.066

As shown in Table 4.11, within input group, the depreciation expense is assigned the largest weight in both 2009 and 2010 while "Other Expenses" gets the lowest weight. By review the output group, the "Passenger Volume" is distributed with the highest weight, while "Satisfaction Index" is assigned the lowest weight. With the cones generated from Fuzzy-AHP model, the revised DEA model is able to reflect preference information over selected input and output when assessing bus operator efficiency.

4.3.4.2 Efficiency Evaluation with the Constrained Cones

After implementation of the aforementioned steps, two optimized input and output pair-wise matrices with their max eigenvalues are obtained to generate the constraint cones, which are ready for adding into the DEA model. Evaluation results are summarized in Table 4.12.

Table 4.12 Efficiency Result of Proposed Model

Bus Operators	2009 Efficiency	2009 Ranking	2010 Efficiency	2010 Ranking
Nanjing	0.810	6	0.760	5
ZhongBei	0.966	2	0.915	2
YaGao	0.916	3	0.832	4
XinCheng	0.874	5	0.732	6
XinNingPu	1.000	1	1.000	1
PuKou	0.911	4	0.894	3
LiuHe	0.769	7	0.702	7

4.3.4.3 Comparison and Discussion

Later on, the conventional DEA model is also employed to measure the relative efficiency which is used to make a comparison with the proposed model. Table 4.13 descripts the result.

Table 4.13 Comparison between the proposed model and the conventional DEA

Bus Operators	2009 DEA	2009 Enhanced-DEA	2010 DEA	2010 Enhanced-DEA
Nanjing	1.000	0.810	0.955	0.760
ZhongBei	1.000	0.966	1.000	0.915
YaGao	1.000	0.916	0.953	0.832
XinCheng	1.000	0.874	0.845	0.732
XinNingPu	1.000	1.000	1.000	1.000
PuKou	1.000	0.911	1.000	0.894
LiuHe	1.000	0.769	1.000	0.702

Table 4.13 clearly demonstrates that conventional loses function in identifying the difference of bus operators since all companies are assessed to be efficient based on 2009 dataset. In contrast, results generated from proposed model shows that only XinNingPu remains efficient when taking into account preferences information over indicators. LiuHe experiences a significant decline from 1.000 to 0.769 because of a relatively poorer performance in passenger volume and operated mileages that are assigned weights, 0.575 and 0.352 respectively. PuKou is another interesting case as the enhanced DEA has modified its efficiency from 1 to 0.894 by reason of a poor performance in "passenger volume". It is noted that PuKou obtains a higher score in prospective of fuel cost control, however, it exerts a less impact in efficiency assessment than "passenger volume" due to a lower weight.

Standing on the dataset of 2010, ZhongBei, XinNingPu, PuKou and LiuHe are all evaluated as efficient units by the conventional DEA model, however, three of them, ZhongBei,,PuKou and LiuHe are assessed to be not efficient anymore by the Enhanced DEA model. There is a reason to believe the change is caused by the add-in of constraint cones. In this case, the labor cost and the depreciation expenses in input group (0.287 and 0.305) as well as the passenger volume (0.582) in output group show higher weights over others which suggests more contributions to the efficiency evaluation. Consequently, because of a relatively poorer performance in those three aspects, ZhongBei, PuKou and LiuHe are justified to inefficient units via enhanced model. In other words, the result also suggests the improvements to labor cost, depreciation expense and patronage.

Regarding the case of XinNingPu who reaches efficient status in both models of both years, it demonstrates a relative balanced and outstanding achievement in all selected criteria with no obvious deficiency.

By comparing the performance of seven bus operators in year 2009 and 2010, both conventional DEA and enhanced DEA reveal that NanJing, YaGao and XinCheng experience a decrease trend in efficiency assessment. However, by contrast to a decline in efficiency identified by proposed model, ZhongBei, XinNingPu PuKou and LiuHe are suggested to remain their efficient position by conventional DEA. The reason could also be contributed to the introduction of weights over indicators. Taking ZhongBei as an example, its depreciation expense has increased from 42.19 million RMB in 2009 to 47.93 million RMB in 2010 while the patronage volume decreased from 215.05 million to 208.52 million. However, the conventional DEA is unable to detect those negative influences because of a weight-free assumption while the proposed model successfully targets the adjustments and takes them into consideration via an introduction of constraint cones.

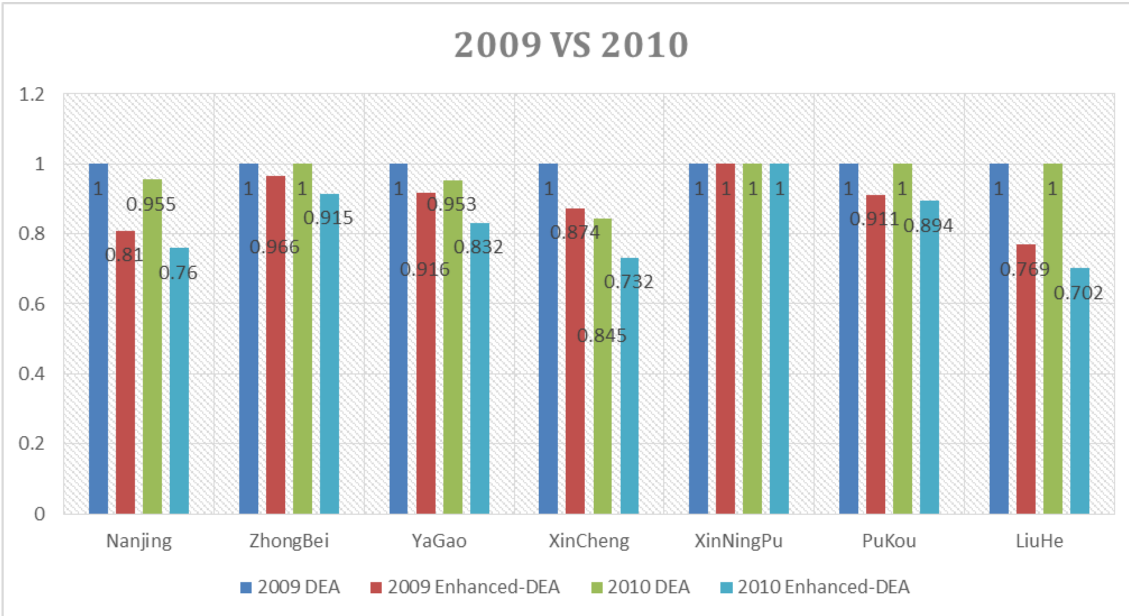


Figure 4.4 Comparison between 2009 and 2010

In addition to yielding the overall ranking for all cities, the implementation of the Fuzzy-AHP model can also generate scores for each operator corresponding to any specific indicator. The feature is expect to help operator to identify their weaknesses and deficiency.

4.3.4 Conclusion

This section presents an enhanced Data Envelop Analysis (DEA) model, which modified conventional DEA model by adding the constraint cones generated from the Fuzzy-AHP model to evaluate transit operator's efficiency. The proposed model aims at including preference information over indicators into DEA process. The new model is designed to effectively solve a biased assumption of conventional DEA that no output or input is more important than the others as well as offering the advantages in ranking those efficient units. An extended Fuzzy-AHP model is employed to generate the constraint cones, which could prevent the vagueness and uncertainty. The characters of new system are applicable to help Bus Company identify its technical efficiency of input resource utilization.

To illustrate the applicability of the proposed approach, a real case in Nanjing City, the capital of Jiangsu province has been has been selected where the efficiencies of seven bus companies are assessed based on 2009 and 2010 dataset. A comparison between conventional DEA and enhanced DEA is also unfolded to clarify the new system's dominance. Results reveal that the proposed model is more applicable in evaluating the transit operator's efficiency and encouraging a boarder range of applications.

4.4. Route-level Transit System Efficiency Assessment

4.4.1 Research Motivation

Similar to bus operator-level efficiency assessment, bus routes are often treated as production lines to compare multiple inputs and outputs when assessing the efficiency of bus routes (Fare and Grosskopf, 1996, 2000; Seiford and Zhu, 1999; Boile, 2001; Nolan *et al.* 2002; Sexton and Lewis, 2003; Zhu, 2003; Karlaftis, 2004; Nakanishi and Falcocchi, 2004; Hwang and Kao, 2006; Tsamboulas, 2006; Barnum *et al.*, 2008; Kao and Hwang, 2008; Sheth *et al.*, 2007; Lao and Liu, 2009; Sanchez, 2009; Yu and Fan, 2009; Lao and Liu in 2009; Zhao *et al.*, 2011; Hawas *et al.*, 2012; Karlaftis and Tsamboulas, 2012; Hawas *et al.* 2012). In this regard, as the widely valuable method, Data Envelopment Analysis (DEA), a non-parametric method introduced by Farrell (1957) and popularized by Charnes *et al.* (1978), is also usually the first-choice by the majority of researchers. However, unlike bus operator performance evaluation, bus routes are always demanded a much more frequent commands to report their efficiency or evaluate their performance, and it directly leads to a shorter evaluation period by comparing with Bus Company-level efficiency assessment. Under such circumstance, any imperfect or unrepresentative dataset may generate a biased efficiency estimation. Furthermore, since the traditional fixed point estimation technology is unable to reflect efficiency fluctuation, it is difficult to be utilized to monitor and supervise bus routes' performance by transit authority.

From literature review, some critical issues of previous applications with using DEA in transit routes efficiency evaluation have not sufficiently investigated. For example, DEA evaluates the relative efficiency of decision-making units but does not allow for ranking of the efficient units themselves (Charnes et al., 1978; Andersen and Christian, 1993; Cook and Seiford, 2009), which has been widely recognized as the major weakness of DEA model. Further, some statisticians and economists have stated that although DEA has some incomparable advantages of parameter estimation, it will lead to the deviation of efficiency evaluation in the case of small samples (Korostelev *et al.* 1995; Simar and Wilson, 1998; Song *et al.* 2013); therefore the reliability of evaluation results could be potentially degraded by unrepresentative data sample. In addition, DEA ignores the inevitable variation of efficiency of decision-making units. Most importantly, DEA calculations are traditionally value-free and the underlying assumption is that no output or input is more important than the other, although in the real world there often exists different importance over different input or output indicators (Halme et al., 1999).

To remedy such limitations and propose a capable model for bus route-level efficiency assessment, this research aims to develop a new approach with sufficient capability and reliability to handle imperfect data and variation of efficiencies due to judgment mistakes and measurement errors. More specifically, this section will develop a Super DEA model to aggregate various route-level transit performance indicators into one comprehensive index for ranking and comparison. A Bootstrap method is further developed to convert the point estimation of efficiency into interval-based estimation of efficiency to improve efficiency estimation reliability.

4.4.2 The Proposed Model

4.4.2.1 Selection of Evaluation Indicators

The proposed transit route efficiency assessment model features a DEA framework, in which a set of representative indicators associated with transit route's performance have been selected and classified into two groups, the input group and the output group. The input group includes those indicators that are associated with investing public resources for transit operation, for example, fuel cost, labor cost and depreciation expenses, etc. The output group of indicators may include the volume of patronage, passenger mileage, and passenger satisfaction index. Note that the proposed evaluation framework and the model offer the flexibility to accommodate other evaluation indicators depending on data availability.

4.4.2.2 The Bootstrap Super-DEA Model

This study has extended the super-DEA model developed by Andersen and Petersen (1993) for transit efficiency evaluation, given by:

$$Min\ \theta - \varepsilon(\textstyle\sum_j^n s_j^- + \sum_j^n s_j^+) \quad (1)$$

$$s.t. \textstyle\sum_{j=1, j\neq j_0}^n \lambda_j x_j + s_j^- = \theta x_{j0} \quad (2)$$

$$\textstyle\sum_{j=1, j\neq j_0}^n \lambda_j y_j - s_j^+ = y_{j0} \quad (3)$$

$$\lambda_j, s_j^-, s_j^+ \geq 0 \quad (4)$$

where x_j is an m-dimensional input vector and y_j is an s-dimensional output vector for the j_0^{th} unit; s_j^- is an m-dimensional slack variable vector for input variables while s_j^+ is an s-dimensional slack variable vector for output variables; θ is a scalar defining the share of the j_0^{th} DMU input vector which is required in order to produce the j_0^{th} DMU output vector within the reference technology; λ is an intensity vector in which λ_j denotes the intensity of the j_0^{th} unit; ε is a non-Archimedean infinitesimal.

In Super-DEA model, the objective function stands for the environmental efficiency value, which needs to be measured. The slack variables of input, s_j^- represents how much *jth* input can be reduced when the DMU0 reaches the production frontier. While s_j^+ represents how much *jth* output can be increased when DMU0 reaches the production frontier. Super-DEA model breaks the tie in efficient units which allows efficiency value to be bigger than 1. Specifically, the efficiency of being less than 1.0 reveals that the highest efficiency is not achieved for the current input and output, the efficiency of being equal to 1.0 represents the best performance, and a greater than 1.0 efficiency indicates an over-utilization of input.

The super efficiency model is able to re-rank the DMUs whose efficiency values are all equal to 1.0 and identify the input redundancies of the DMUs whose efficiencies are under the highest level by introducing the slack variable. The model has successfully broken the tie between efficient units in tradition DEA models (Lei, 2007; Wei et al., 2012; and Song et al., 2013).

To further address the variation of efficiencies in case of small and unrepresentative samples and prevent the errors due to judgment mistakes, this study has integrated a statistical resampling method, Bootstrap, with the super-DEA model to yield confidence intervals of efficiency estimation (Simar and Wilson, 1999).

The basic idea of Bootstrap Super-DEA method is to make a numerical simulation of the original sample data, and to conduct super DEA efficiency calculation for a large number of produced simulated samples. The procedures can be summarized as follows (Simar and Wilson, 1998, 1999, 2000; Maghyereh and Awartani, 2012; Song et al., 2013):

1. Use super-DEA model to obtain the efficiency scores $\widehat{\theta_i}$, for each bus route $i = 1, \dots, n$, by solving the Eq.(1) –Eq.(4);
2. Simulate the smoothed Bootstrap sample $\tilde{\theta}_1^* \dots \tilde{\theta}_n^*$, for $i = 1, \dots, n$, by applying the following formula:

$$\tilde{\theta}_i^* = \begin{cases} \beta_i^* + h\varepsilon_i^* & if\ \beta_i^* + \varepsilon_i^* < 0 \\ 2 - \beta_i^* - h\varepsilon_i^* & otherwise \end{cases} \quad \varepsilon_i^* \sim N(0,1) \tag{5}$$

where β^* is a non-smooth sample generated with replacement from $\widehat{\theta_1}, \dots, \widehat{\theta_n}$; h is the bandwidth of a standard normal kernel density, and ε_i^* is a draw from an *iid* standard normal. An obvious problem in any smoothing procedure is the choice of the bandwidth of the density estimate h. In the procedure performed in this paper, we follow a robust bandwidth selection rule that yields the lowest mean integrated squared error (Simar and Wilson, 2004);

3. Obtain the corrected smoothed bootstrap sample (pseudo efficiencies $\theta_n^* \ for\ i = 1, \dots n,$) using the following formula:

$$\theta_i^* = \overline{\beta^*} + \frac{\tilde{\theta}_i^* - \overline{\beta^*}}{\sqrt{1 + h^2/\sigma_\theta^2}} \tag{6}$$

where $\overline{\beta^*}$ is the average of the re-sampled efficiencies, given by,$\overline{\beta^*} = (1/\mathrm{n}) \sum_{i=1}^{n} \beta_i^*$, and σ_θ^2 is the variance estimate of the measured efficiencies $\tilde{\theta}_1^* \dots \tilde{\theta}_n^*$. The corrected efficiency is introduced to ensure the convergence of the bootstrapped efficiency;

4. Compute the pseudo-variable inputs $\{(x_{ib}^*, y_i), i = 1, \dots, n\}$ by applying the ration formula:

$$x_{ib}^* = (\hat{\theta}_\iota / \theta_{ib}^*) x_i \tag{7}$$

5. Apply the pseudo-variable inputs into the super-DEA model to compute the Bootstrap Super-DEA efficiency $\widehat{\theta_\iota^*}$ for each bus route;
6. Repeat steps 2-5, B times to obtain B robust efficiency scores $\widehat{\theta_{\iota b}^*}, b = 1, \dots, B$;
7. Calculate the bias-corrected estimator of original efficiency scores θ_i as follows:

$$\widehat{\theta_\iota^*} = 2\hat{\theta}_\iota - B^{-1}(\textstyle\sum_{b=0}^{B} \widehat{\theta_{\iota b}^*}) \tag{8}$$

8. Determine the confidence interval at α level by using the empirical distribution of the bootstrapped efficiencies (Tortosa-Ausina et al. 2009, 2010.

Firstly, we sort the values $\{\widehat{\theta_{\iota b}^*}, b = 1, \dots, B\}$, in increasing order and then delete $(\frac{\alpha}{2} * 100\%)$of the elements at either end of the sorted set.

Secondly, we select $a_{i,\alpha/2}\ and\ a_{i,1-\alpha/2}\ i = 1, \dots n$; to represent two endpoints of the sorted array respectively, the approximated confidence interval for each bus route is:

$$\widehat{\theta_\iota} - a_{i,1-\frac{\alpha}{2}} < \theta_k < \widehat{\theta_\iota} + a_{i,\frac{\alpha}{2}} \, i = 1, \ldots, n; \tag{9}$$

As following the above steps, the Bootstrap method is able to get the distribution of the original sample estimator, and further to correct biased estimates of the efficiency value in case of small sample data. In addition, the obtained efficiency boundaries can best assist transit operators to identify problematic bus routes with extremely low operational efficiencies or over-utilization of available resources.

4.4.3 Case Study

This study has selected 17 routes from the 3rd bus company in Chongqing, China for case studies. With an area of 82,400 square kilometers (31,800 square miles) and the population of 30 million, Chongqing is the biggest municipality (in terms of area and population size) under direct administration by the Chinese central government. By the end of 2012, more than 8000 buses were at service and 500 bus routes were operated by eight bus companies. The selected 17 routes all operate in the main urban corridors and are managed by the 3rd bus company, the largest operator in Chongqing city. With data collected AM peak period of the year of 2012, this study applies the proposed model to evaluate the operational efficiencies of the selected 17 routes and compares the results with the conventional DEA model and the Super-DEA model.

4.4.3.1 Data

Data used in this study were collected from the operational report of the 3rd Chongqing Bus Company. The data is specifically processed for the annual average of the peak period (7:45am to 8:45am) which is one of most concerns by transit operators. Data collected and processed are summarized in Table 4.14. In the proposed DEA model, operation cost and total capacity are classified as input indicators as both of them are resources invested to maintain transit service while mileages and passenger volume are grouped as the output indicators.

Table 4.14 Data collected for case study

Bus Routes	Operation Cost (Yuan)	Total Cap (Person)	Mileages (Vehicle*Km)	Passenger Volume (Trip)
301	3119.55	2790	439.65	3792
308	2888.48	1540	420.64	1704
318	3669.12	2450	537.16	3377
319	1748.46	1540	276.78	2410
325	2006.55	1820	268.65	1401
338	1580.46	1000	223.78	1931
346	1374.66	1190	215.38	2131
349	2551.50	2660	373.5	2693
354	2864.40	1750	394.2	4074
362	1918.35	1540	296.05	2583
363	1704.78	1863	268.54	3079
364	2978.85	2450	408.55	3801
365	1587.18	1020	255.74	2158
368	1040.20	1120	172.6	1410
372	1411.20	1190	210.6	1171
381	603.40	600	74.2	1551
382	837.41	700	130.63	680

As shown in Table 4.14, one can observe that the operation cost of AM peak period ranges from 603.4 RMB (Route 381) to 3669.12 RMB (Route 318) while the total supplied capacity varies from 600 (Route 381) to 2660 (Route 349). In the meantime, the operated mileages locates in range 524.16 km (Route 318) to 86.2 (Route 318) while the Route 354 carries 4074 passengers which leads to others and the total ridership of Route 382 is only 1/6 of Route 345 which ranks final. The observation from data dump generally shows a reasonable and explanatory positive relationship between input and output groups, more input raise more output generate. However, some unusual and remarkable cases are worth to pay more attentions, for example, the total supplied capacity of Route 325 is 400 which is more than its demand while its cost per passenger is also relative higher that other routes. Another opposite case is Route 381 which its total passenger volume is more than double of its capacity while its operation cost per passenger is the lowest during the all selected routes.

4.4.3.2 Bus Routes Efficiency Assessment by Conventional DEA

The case study firstly employs the conventional DEA to assess the bus routes' efficiencies. The adopted CCR model was proposed by Charnes *et al.* in 1978. The result is listed in Table 4.15.

The result shows that all the units are assessed to be efficient because all the routes have fully utilized their input resource measured by conventional DEA model. Obviously, the tie among all the efficient units is unable to break and the deficiency of DEA leads to a dysfunction in evaluating the performance of selected 17 bus routes in Chongqing.

Table 4.15 Bus routes' efficiencies by Conventional DEA and Super-DEA

Bus Route	DEA Score	Super-DEA score
301	1	0.8647
308	1	1.0656
318	1	0.8956
319	1	0.9733
325	1	0.8079
338	1	0.9024
346	1	0.9829
349	1	0.8806
354	1	1.0346
362	1	0.9534
363	1	1.0343
364	1	0.8543
365	1	1.1198
368	1	1.0247
372	1	0.9078
381	1	1.4876
382	1	0.9514

4.4.3.3 Bus Routes Efficiency Assessment by Super-DEA

To mitigate the situation, the case study then introduces the proposed Super-DEA model to achieve the task of evaluating the bus routes' performance by solving the Eq. (1) to Eq. (4). Table 2 records the result.

By implementing proposed Super-DEA model, the tie between efficient units are broke by introduction of the slack variables. Specifically, the efficiencies of Route 301, 318, 319, 325, 338, 346, 349, 362, 364, 372 and 382 are adjusted to be lower than 1 indicating current inputs have not been fully utilized while the rest are assessed to be super-efficient revealing an overdevelopment on current supplies. Although the units' efficiency is successfully distinguished, the proposed Super-DEA model remains the issue of ignorance to statistical test and unable to provide the statistical distribution of efficiency score. Furthermore, the interference of bias and error from the issue of same data sample still exists.

4.4.3.4 Bus Routes Efficiency Assessment by Bootstrap Super-DEA

To take the statistical prospective into account, the proposed Bootstrap method is applied to modify the efficiency derived from Super-DEA by processing Step 1 to 8 and Eq. (5) to Eq. (9). Noticeably, in this case, the value of B is set to 2000, indicating 2000 Bootstrap samples will be manipulated to generate Bootstrap efficiency, as 2000 iterations are suggested to ensure adequate coverage of the confidence intervals by Simar and Wilson in 2000 and Tortosa-Ausina *et al.* in 2012. Table 4.16 and Figure 4.4 record a comparison result among conventional DEA, Super-DEA and Bootstrap Super-DEA models. Figure 4.5 presents the efficiency interval obtained from proposed model.

Table 4.16 Comparison in Conventional DEA, Super-DEA and Bootstrap DEA

Route	Conventional DEA	Super-DEA	Corrected Efficiency Value	Lower Bound	Upper Bound	Variance
301	1	0.8647	0.7163	0.6489	0.8054	0.0014
308	1	1.0656	1.0784	0.9733	1.2442	0.0037
318	1	0.8956	0.8313	0.7319	1.0333	0.0045
319	1	0.9733	0.8382	0.7785	0.8915	0.0008
325	1	0.8079	0.5073	0.4410	0.5634	0.0009
338	1	0.9024	0.7041	0.6378	0.7722	0.0012
346	1	0.9829	0.8370	0.7797	0.8927	0.0008
349	1	0.8806	0.6863	0.6173	0.7449	0.0010
354	1	1.0346	1.0391	0.9297	1.2534	0.0050
362	1	0.9534	0.8114	0.7493	0.8681	0.0008
363	1	1.0343	1.0318	0.9311	1.1762	0.0035
364	1	0.8543	0.6782	0.6024	0.7659	0.0015
365	1	1.1198	1.1369	1.0555	1.2131	0.0016
368	1	1.0247	0.8782	0.8169	0.9634	0.0012
372	1	0.9078	0.6757	0.6139	0.7351	0.0009
381	1	1.4876	1.7114	1.5872	1.9713	0.0066
382	1	0.9514	0.7067	0.6306	0.8638	0.0025

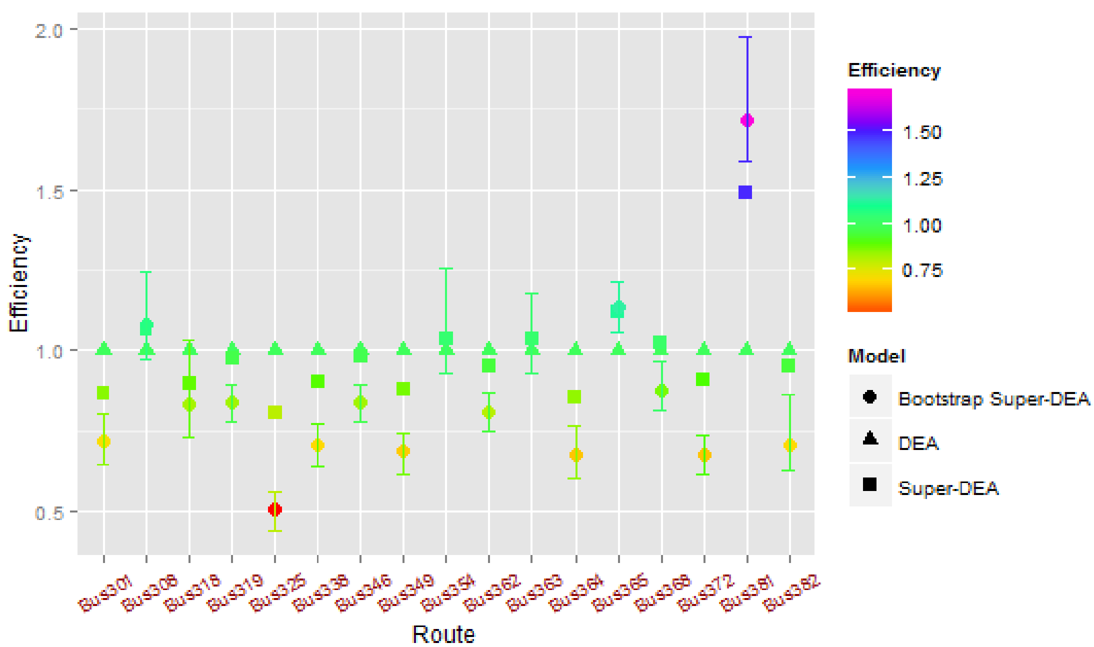


Figure 4.5 Bus route Efficiency assessment by DEA, Super-DEA and Bootstrap Super-DEA

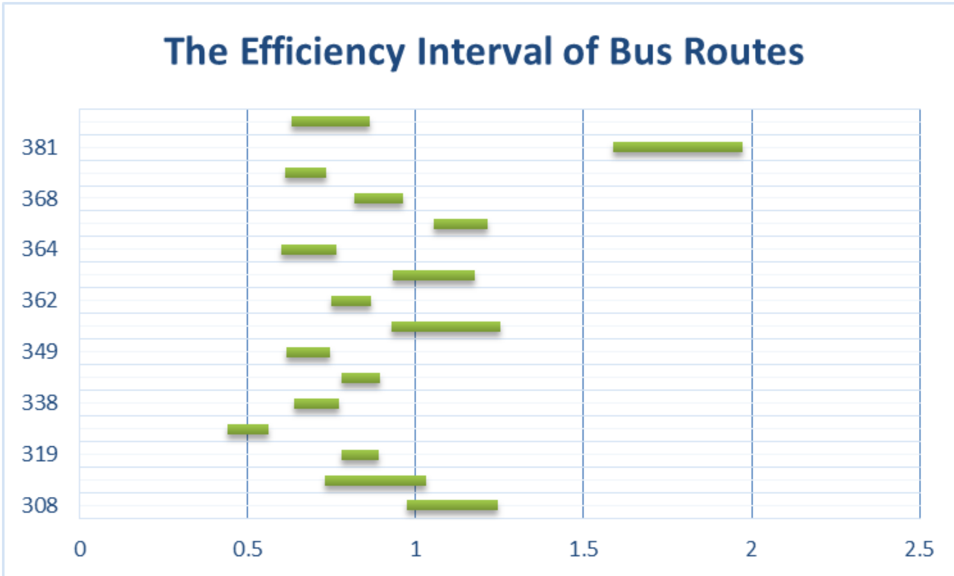


Figure 4.6 Efficiency Interval

After implementation of Bootstrap, the efficiency obtained from Super-DEA is further modified according to iterative results of Bootstrap process. The model also generate the 95% confidence interval of efficiency value for each bus route which can be a benchmark to monitor the performance of bus routes.

4.4.3.5 Discussion and Analysis

This case study firstly utilizes a conventional DEA to assess the selected 17 bus routes. The result shows all the units were evaluated to be efficient indicating the conventional DEA failed to identify the difference in all involved bus routes due to its theoretical limitation in ranking those efficient units. The Super-DEA is therefore proposed to improve DEA model's performance by the introduction of slack variables. The results suggests an obvious progress of distinguishing those efficient routes into inefficient or super-efficient, specifically, route 308, 354, 363, 365, 368 and 381 are assessed to be super-efficient units due to their excessive utilization on current supplies. The efficiency of Route 381 leads to others which due to its much higher demand over capacity (Passenger Volume/Total Capacity =2.58) as well as its lowest unit passenger cost (Total Cost/Passenger Volume=0.39). It is found that the similarities between super-efficient bus routes are concluded as: 1) the passenger demand is higher than the supplied capacity; 2) the unit cost is relatively lower than under-efficient units. From the transit operators' prospective, all of the facts indicate a high-utilization on current supplies which will increase companies' profit and maximum the utility of their invested resources. In the meantime, from the standpoint of the public, the super-efficiency may also imply an over-crowed condition for those bus routes and further leads to an uncomfortableness to passengers. For those under-efficient bus routes, they have not perfectly used resources efficiently. For example, Route 382 ranked last because of a lowest passenger volume/total capacity ratio, 0.77, as well as a relatively higher unit cost (1.43 yuan/passenger).

To further introduce statistical test into DEA analysis, a Bootstrap method is following applied to modify the efficiency derived from Super-DEA model as well as providing the corresponding confidence interval. As can be seen from Table 4 and Figure 1, the corrected efficiency values of the most cases are smaller than Super-DEA efficiency values, which means that the results of those bus routes' efficiency as calculated by Super-DEA is overestimated which mainly because of only 17 samples. However, the super-efficient units of Route 365, 308 and 381 are modified to be more efficient. After Bootstrap correction, the efficiency value is changed as well as a change in their efficiency ranking. Route 372 jump from 11st to 16th while Route 301 up from 15th to 11st. As the traditional DEA method is strongly dependent on the data and the statistical characteristic is unable to be estimated when the number of samples is few, so that the calculated efficiency may be biased and may not be strong enough to represent the real situation. From the deviation value, the greater original efficiency value, the greater absolute deviation generates after correction. These deviations reflect the accuracy of original efficiency values. The greater the deviation is, the lower the accuracy is. In our case, the accuracy of estimator generated from Super-DEA is acceptable due to the usage of the whole year average data source.

To verify the process of bootstrap, the proposed model also produces variance collected from 2000 iterations. As we can see from the table, all the variances are smaller than 0.01 which describes a reliable process with a slight fluctuation.

From the confidence interval, there exists a span of the upper and lower bounds for each involved bus route, indicating its reasonable fluctuation space. It could be regarded as a benchmark or reference for manager to monitor and control the operation process, by which the manager is able to target and draw immediate attentions to those bus routes whose efficiency is blew the lower bound.

The introduction of bootstrap method not only offers statistical analysis to DEA model, but also improves accuracy of efficiency estimation.

4.4.4 Conclusion

DEA, as a popular and sought-after method in evaluating transit system efficiency, is suffering a number of deficiencies. When using DEA to evaluate transit system efficiency at route-level, some critical issues need to be further investigated, which are: 1) a tie in efficient units, and 2) an ignorance of statistical test. Both of them place some risks of generating biased and unrepresentative efficiency scores, especially in case of a small sample dataset. In addition, conventional fixed point estimation methodology with using conventional DEA model is incapable to capture the fluctuation of efficiency which results in a barrier to promote the relevant methodologies into real world practice. Realizing such deficiencies, the objective of this research contributes to filling the vacancy of a Bootstrap-Super DEA model with sufficient capability to remedy the limitations of conventional DEA model, and further apply the proposed model to evaluate and monitor transit system performance at route level.

A super-DEA model is firstly designed to assess the bus routes' efficiency by which the theoretical defect of lacking capability to rank those efficient units in conventional DEA is efficiently solved. A following step, the Bootstrap method, is applied to modify the efficiency derived from super-DEA model as well as generating the efficiency distribution and taking statistical test into account. After the implementation of Bootstrap method, a corrected efficiency value and the corresponding confidence interval are offered. The obtained interval is further considered as the benchmark and reference for manager to monitor and control the transit operation. To illustrate the usefulness and usability of the approach, a real case in Chongqing Metropolitan, China has been summarized to evaluate 17 bus routes' efficiency. A comparison between conventional DEA, Super-DEA and Bootstrap Super-DEA with detailed discussions is unfolded to clarify the new model's functions. Results reveal that the proposed model is more applicable in evaluating the transit operator's efficiency and encouraging a boarder range of applications.

Chapter 5: Incentive-based Subsidy Allocation Model

5.1. Introduction

Due to the fact that traditional capital-based or cost-proportional (e.g. total mileage, fuel consumption, or total passenger-trips) methods always produce a negative correlation between the amount of capital-based subsidy and the performance of transit operators (Obeng and Sakano, 2008), development an incentive-base subsidy allocation mechanisms becomes an essential task to allocate subsidies to cover their operational loss as well as to encourage them to provide better services in the next operational cycle. The new system is expected to properly integrate the operational performance or efficiency of transit industry into the subsidy allocation process. In review of relevant literature, very limited studies have linked efficiency evaluation with the subsidy allocation at bus operator level, resulting in lack of effective framework and methodology for incentive-based subsidy allocation. In addition, insufficient efforts have been made in developing a practicable model and applicable framework for route-level subsidy allocation in the literature.

Realizing such deficiency of existing studies and importance of the proposed topic, this chapter contributes to filling the vacancy of a theoretically justified model in literature that can allocate limited subsidies to urban transit operators according to their operational and financial efficiencies. Furthermore, a centralized resource allocation model will be developed to subdivide a company's subsidy and targets into its subordinated bus routes. At first, this chapter will develop a bus operator-level incentive-based framework, consisting of key modules of baseline assessment of transit operational and financial efficiency, efficiency-based target setting and pre-evaluation, incentive-based subsidy allocation, as well as feedback and subsidy adjustment. Note that a robust enhanced slack-based measure (SBM) of super efficiency DEA model is introduced to assess bus operators' financial efficiencies. The model features the use of "slacks" to represent the cost excess and efficiency shortfalls, and to deal with them directly by maximizing operators' slacks. The subsidy assignment is relied on an extended inverse DEA model according to their operational and financial efficiencies. The complete process factors in identifying the amount of subsidy based on the operators' improvements of operational and financial efficiency as well as preserving units' financial efficiency. After an incentive-based subsidy allocation for bus operators, a centralized resource allocation and target setting model is developed to distribute fixed subsidies and set targets to bus routes simultaneously. The application of new system aims to optimize input resources utilization and further improve bus routes' efficiencies. Eventually, two convincing cases are illustrated for the proposed framework and models which assist government or transit managers and authorities in best understanding and applying the proposed models during the process of transit subsidy allocation.

5.2. An Incentive-based Subsidy Allocation Framework

This section will illustrate the modeling framework of the proposed incentive-based subsidy allocation process from operator-level to route level and the interrelations between its principle components.

Figure 5.1 depicts the framework of the proposed system for allocating subsidy at operator-level and route-level, highlighting interrelations among efficiency assessment module introduced from **Chapter 4**, financial assessment module and incentive-based subsidy allocation module.

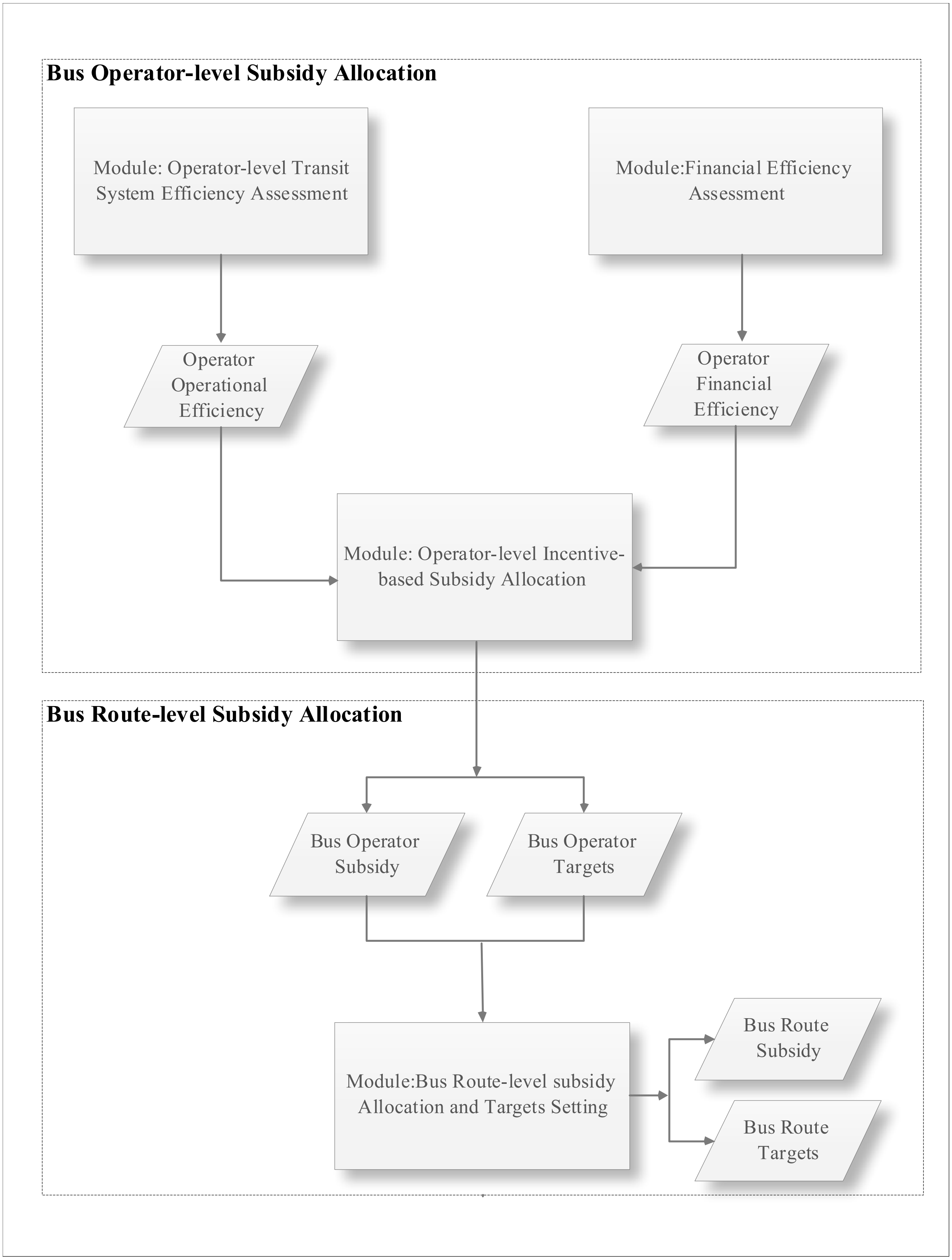


Figure 5.1 A framework of incentive-based subsidy allocation process

5.3. Bus Operator-level Incentive-based Subsidy Allocation Model

5.3.1 Research Motivation

In review of existing literature, it is ascertained several challenges when allocating subsidy to bus operators, which are:

- Traditional cost-based subsidy mechanisms is hard to exert positive influences in improving bus companies' performance, especially from the service quality prospective;
- Bus companies themselves are always excluded from the process of subsidy allocation;
- Budgetary uncertainty is a significant concern which places the financial risk on government or funder;

To account for those problems, a comprehensive evaluation and decision framework is developed in process of operator-level incentive-based subsidy allocation, consisting of key modules of baseline assessment of transit operational and financial efficiency, efficiency-based target setting and pre-evaluation, incentive-based subsidy allocation, as well as feedback and subsidy adjustment. When evaluating the efficiency of transit operators, criteria including fleet size, human resource, volume of patronage, mileage, and passenger satisfaction index, etc. are considered and properly weighted into the Operator-level Transit System Efficiency Assessment model introduced in Chapter 4.3. In the meantime, an extended slack-based measure of super efficiency DEA model is developed to evaluate operator's financial efficiency, criteria including fuel cost, labor cost, and depreciation expense, etc.

Using the above obtained financial and operational efficiencies as the new set of outputs and the operational cost as the new set of inputs, an extended inverse DEA model is developed to allocate incentive-based subsidy. The new system allows each transit operator to set the target output and input levels according to its operational constraints and capabilities. The corresponding improvements are then processed by the inverse DEA model to identify extra inputs to each transit operator (i.e. the subsidy allocated). Therefore, allocation of the subsidy mainly depends on the improvement of a transit operator's performance rather than the running cost. Noticeably, a procedure of subsidy adjustment between the target-based efficiency and the actual efficiency is required to act after a collection of real operational and financial data by the end of the next operational year. The detailed procedure is described in Figure 5.2.

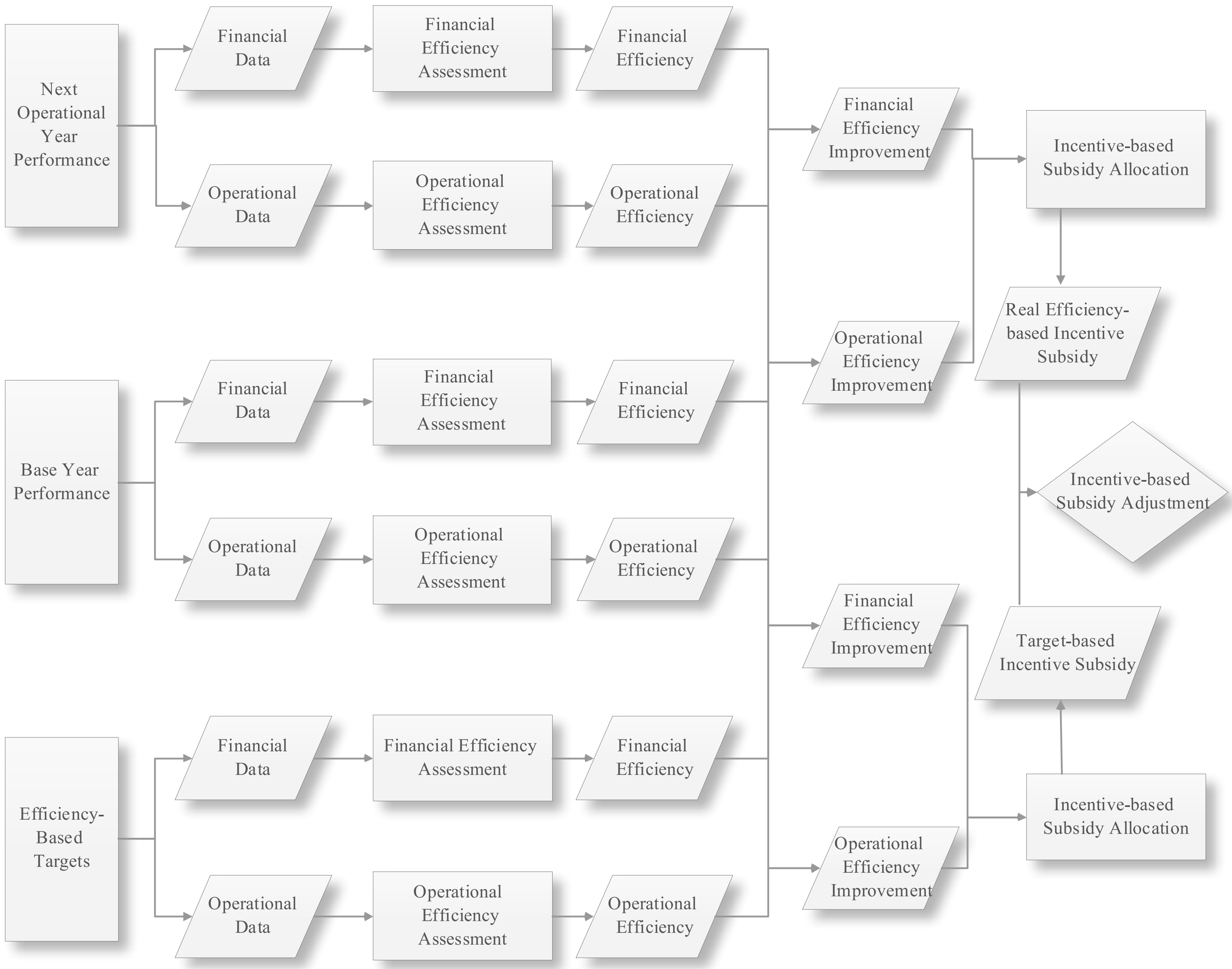


Figure 5.2 Bus operator-level incentive-based subsidy allocation model framework

5.3.2 Operator-level Transit System Operational Efficiency Assessment Model

As a natural extension of previous study of transit system efficiency assessment at bus operator level, the module of operator-level transit system efficiency assessment adopted in this section follows the proposed model from Chapter 4.3, which is the enhanced Data Envelopment Analysis (DEA) model with constrained cones. The developed model factors in introduction of preference cone constraints into the DEA model which is critical for decision makers to incorporate their preferences or important policies over inputs/outputs into the performance evaluation and subsidy allocation process. A Fuzzy-AHP model is developed to tackle the preference determination problem. Different from the conventional AHP, the proposed model adds a fuzzy scale level between the criteria level and the alternative level, which offers the advantage of preventing the vagueness and uncertainty on judgments of decision-makers. Such a unique modeling feature is further embedded with a non-linear optimization formulation to maximize the consistency in pair-wise comparison and importance estimation for each criterion. With the preference cone constraints, the DEA model offers the capability to compare the performance of transit operators with identical efficiency score of 1.0 when the standard DEA approach is employed.

The module of operator-level efficiency assessment functions in evaluating bus companies' base line operational efficiencies, target-based operational efficiencies and next year operational efficiencies. When assessing operational efficiency, the input indicators only focus in operational prospective, likely, bus fleet size, human resources and consumed fuel etc., exclude the cost information which will be introduced into financial efficiency assessment module in the later section. The obtained efficiency scores are then selected as one of the key input data for incentive-based subsidy allocation model.

5.3.3 Operator-level Transit System Financial Efficiency Assessment Model

5.3.3.1 Introduction

The report of World Bank (2014) has clearly stated that "Public transport systems have to balance financial sustainability with a need to provide affordable services." Since this dissertation introduces a module of efficiency-based target setting, it allows operators to get involved in the process of subsidy allocation as well as enhancing the strength between associated authorities and enterprise own strategic target development. During the whole process, one of the most essential factors of leading incentive-based subsidy allocation to success is preserving bus operator's financial sustainability, which can find its best explanations from TRB report (2004) "A sustainable transportation system will have accountability in the planning process. Performance measurements and feedback loops will enable planners to learn from past experiences and understand fully the ramifications of decisions on the components of sustainability." Especially, with public transit budgets across the country becoming anemic, there is no time better than the present to develop programs and or initiatives that will not only meet the current transportation needs, but also allow for future expansion.

Therefore, a solid and effective financial assessment model to keep the financial healthy becomes a vital part in subsidy program development.

5.3.2.2 The Proposed Model

In this section, a financial efficiency assessment model is proposed to evaluate operator's financial performance by comparing the level of cost input with the passenger services data. It aims to build a linkage between monetary operator's cost and operation efficiency. The model functions in preserving operator's financial sustainability when the incentive-based subsidy allocation model is working to identify extra cost input for reaching efficiency-based targets. To achieve this function, a DEA-based model is selected which is a linear programming (LP) non-parametric technique that evaluates the relative performance of decision making units (DMUs) with respect to multi inputs and multi outputs. A main advantage of DEA is that it does not require any prior assumptions on the underlying functional relationships between inputs and outputs. It is a data-driven frontier analysis technique that floats a piecewise linear surface to rest on top of the empirical observations. In this case, the multi types of cost are considered as inputs of the model while the various operational output data is treated as output data.

Since the advent of data envelopment analysis (DEA), many papers have been published on its methodology and applications. There are two types of DEA models, the radial and non-radial models. The CCR model measures the radial efficiency of the inputs (input-oriented) or outputs (output-oriented) by gauging the ratio of the inputs to be contracted or the ratio of the outputs to be enlarged so that the evaluated decision making units (DMU) becomes efficient. The radial efficiency reveals the existence of excesses in inputs and shortfalls in output at the same time (call slacks) (Tone, 1999). A DMU with full ratio efficiency and no slacks in any optimal solution from DEA model is called efficient. Otherwise, the DMU has a disadvantage against the DMUs in its reference-set. Therefore, one of the limitations of radial models is that radial efficiency does not reflect all inefficiency of a DMU (Morita et al., 2005). Slacks need to be considered simultaneously with radial efficiency to identify the "real" projection of a DMU (Fang et al., 2013).

In light of these issues, recent studies have tried to develop non radial DEA approaches (Tone, 1999; Fukuyama and Weber, 2009; Zhou et al., 2012). Among non-radial efficiency approaches, slacks-based measures (SBM) which was firstly introduced by Tone at 1999 uses the term "slacks" to represent the input excesses and output shortfalls and deals with them directly and by maximizing these slacks. The model directly accounts for input and output slack in efficiency measurements, with the advantage of capturing the whole aspect of inefficiency. Based on SBM, Tone (2002) further developed a slack-based measure of super efficiency DEA model which is able to break the ties among efficient units. This section introduces an alternative approach developed by Fang et al., (2013) to further refine and extend Tone's SBM Super Efficiency Model.

By review the current operational situations of bus companies from both developing and developed regions, different bus operators are obviously varied in their cost structure, input resource scale and output scale which all directly lead to an evident gap of the financial efficiency in efficient units and inefficient units. Under this condition, the neglect of slack variables of conventional radial-based efficiency measures would highly likely results in biased efficiency estimation. Thus, all above mentioned good properties of SBM Super Efficiency DEA model are particularly suitable for transit operator financial efficiency assessment.

According to Tone (2002)'s theory of SBM Super efficiency DEA model, suppose there are n DMUs associated with m inputs and s outputs. Let x_{ij} denote the ith input of DMU j and y_{rj} denote rth output of DMU j. Assume that all data are positive, i.e., $x_{ij}, y_{rj} \geq 0$ for all possible$i = 1, \dots, m; r = 1, \dots, s; j = 1, \dots, n;$

The production possibility set P spanned by all DMUs is defined as:

$$p = \{(x_1, \dots, x_m, y_1, \dots, y_s) | x_i \geq \sum_{j=1}^{n} \lambda_j x_{ij}, i = 1, \dots, m, y_r \leq \sum_{j=1}^{n} \lambda_j y_{rj}, r = 1, \dots, s\} \quad (1)$$

Tone (2002) firstly proposed the following SBM model to evaluate the efficiency of DMU k:

$$\min \rho = \frac{1 - (\frac{1}{m}) \sum_{i=1}^{m} z_i^- / x_{ik}}{1 + (\frac{1}{s}) \sum_{r=1}^{s} z_r^+ / y_{rk}}$$

$$s.t. x_{ik} = \sum_{j=1, j \neq k}^{n} x_{ij} \lambda_j + z_i^- , i = 1, \dots, m;$$

$$y_{rk} = \sum_{j=1, j \neq k}^{n} y_{rj} \lambda_j + z_r^+ , r = 1, \dots, s;$$

$$\lambda_j \geq 0, j = 1, \dots, n, , j \neq k;$$

$$z_i^- \geq 0, i = 1, \dots, m;$$

$$z_r^+ \geq 0, r = 1, \dots, s; \qquad (2)$$

Where z_i^- and z_r^+ are slack variables for input variables and output variables respectively.

According to Tone's theory, the model (2) is firstly used to filter the SBM-efficient units which is defined as the unit with optimal solution $z_i^{-*} = z_r^{+*} = 0$, or $\rho^* = 1$. And then the following model (3) is proposed (Tone, 2002) to discriminate those SBM-efficient units identified by model (2):

$$\min \rho = \frac{(\frac{1}{m}) \sum_{i=1}^{m} \overline{x_i} / x_{ik}}{(\frac{1}{s}) \sum_{r=1}^{s} \overline{y_r} / y_{rk}}$$

$$s.t. \overline{x_i} \geq \sum_{j=1, j \neq k}^{n} x_{ij} \lambda_j , i = 1, \dots, m;$$

$$\overline{y_r} \leq \sum_{j=1, j \neq k}^{n} y_{rj} \lambda_j , r = 1, \dots, s;$$

$$\lambda_j \geq 0, j = 1, \dots, n, j \neq k;$$

$$\overline{x_i} \geq x_{ik}, i = 1, \dots, m;$$

$$\overline{y_r} \geq 0, \overline{y_r} < y_{rk}, r = 1, \dots, s; \qquad (3)$$

However, Fang et al., (2013) stated that the inefficient units cannot be discriminated by model (3) due to the fact that they used model (3) to test those inefficient units identified by model (2), and then the feedback of efficiency was 1. In addition, Tone's model (3) does not incorporate slacks variables explicitly. Therefore, the revised SBM Super Efficiency DEA models are proposed as following:

$$\min \delta = \frac{1 + (\frac{1}{m}) \sum_{i=1}^{m} w_i^- / x_{ik}}{1 - (\frac{1}{s}) \sum_{r=1}^{s} w_r^+ / y_{rk}}$$

$$s.t.\, x_{ik} \geq \sum_{j=1, j\neq k}^{n} x_{ij}\lambda_j - w_i^-, i = 1, \ldots, m;$$

$$y_{rk} \leq \sum_{j=1, j\neq k}^{n} y_{rj}\lambda_j + w_r^+, r = 1, \ldots, s;$$

$$\lambda_j \geq 0, j = 1, \ldots, n, , j \neq k;$$

$$w_i^- \geq 0, i = 1, \ldots, m;$$

$$w_r^+ \geq 0, w_r^+ \leq y_{rk}, r = 1, \ldots, s; \qquad (4)$$

where model (4) and model (3) are equivalent, $\overline{x_i}$ and $\overline{y_r}$ are replaced by x_{ik}, y_{rk} and their corresponding slack variables, w_i^- and w_r^+.

Theorem 1. Model (4) and model (3) are equivalent.

Proof. Substituting $\overline{x_i}$ with $(x_{ik} + w_i^-)$ and $\overline{y_r}$ with $(y_{rk} + w_r^+)$, then:

$$\min \delta = \frac{(\frac{1}{m}) \sum_{i=1}^{m} [\frac{x_{ik} + w_i^-}{x_{ik}}]}{(\frac{1}{s}) \sum_{r=1}^{s} [\frac{y_{rk} + w_r^+}{y_{rk}}]}$$

$$s.t.\, x_{ik} + w_i^- \geq \sum_{j=1, j\neq k}^{n} x_{ij}\lambda_j, i = 1, \ldots, m;$$

$$y_{rk} + w_r^+ \leq \sum_{j=1, j\neq k}^{n} y_{rj}\lambda_j \, , r = 1, \dots, s;$$

$$x_{ik} + w_i^- \geq x_{ik}, i = 1, \dots, m;$$

$$y_{rk} + w_r^+ \geq y_{rk}, y_{rk} - w_r^+ \geq 0, r = 1, \dots, s;$$

$$\lambda_j \geq 0, j = 1, \dots, n, , j \neq k;$$

After a rearrangement, then:

$$\min \delta = \frac{(\frac{1}{m}) \sum_{i=1}^{m} [\frac{x_{ik} + w_i^-}{x_{ik}}]}{(\frac{1}{s}) \sum_{r=1}^{s} [\frac{y_{rk} + w_r^+}{y_{rk}}]} = \frac{1 + (\frac{1}{m}) \sum_{i=1}^{m} w_i^- / x_{ik}}{1 - (\frac{1}{s}) \sum_{r=1}^{s} w_r^+ / y_{rk}}$$

$$s.t. \, x_{ik} + w_i^- \geq \sum_{j=1, j\neq k}^{n} x_{ij}\lambda_j \, , i = 1, \dots, m;$$

$$y_{rk} + w_r^+ \leq \sum_{j=1, j\neq k}^{n} y_{rj}\lambda_j \, , r = 1, \dots, s;$$

$$w_i^- \geq 0, i = 1, \dots, m;$$

$$w_r^+ \geq 0, y_{rk} \geq w_r^+, r = 1, \dots, s;$$

$$\lambda_j \geq 0, j = 1, \dots, n, , j \neq k;$$

So, model (4) and model (3) are equivalent. Model (4) is able to identify the projection in the fourth quadrant of DMU k by minimizing the input slacks (w_i^-) and output slacks (w_r^+). It is worth noting that $y_{rk} \geq w_r^+$ is necessary to ensure that the objective function to be positive. The proposed alternative approach uses model (4) as first step to evaluate units' SBM efficiency, and then, the second model to be applied is illustrated as below:

Let w_i^{-*} and w_r^{+*} denote the optimal solution of model (4). The standard SBM model (2) is revised as follows:

$$\min \delta = \frac{1 - (\frac{1}{m}) \sum_{i=1}^{m} s_i^- / x_{ik}}{1 + (\frac{1}{s}) \sum_{r=1}^{s} s_r^+ / y_{rk}}$$

$$s.t.\, x_{ik} = \sum_{j=1, j\neq k}^{n} x_{ij}\lambda_j - w_i^{-*} + s_i^- , i = 1, \dots, m;$$

$$y_{rk} = \sum_{j=1, j\neq k}^{n} y_{rj}\lambda_j + w_r^{+*} - s_r^+, r = 1, \dots, s;$$

$$\lambda_j \geq 0, j = 1, \dots, n,, j \neq k;$$

$$s_i^- \geq 0, i = 1, \dots, m;$$

$$s_r^+ \geq 0, r = 1, \dots, s; \qquad (5)$$

By applying model (5), the inefficient DMUs can be successfully discriminated which refined Tone's model structure.

Fang et al., (2013) discussed the working philosophy of revised model. In detailed, if DMU *k* is outside the production possibility set spanned by DMUs excluding DMU *k*, model (4) will first identify the minimum distance for DMU *k* from the efficient frontier in terms of the input savings (w_i^-) and output slacks (w_r^+). By adding slack variables to DMU *k*, DMU *k* will be able to stand in the frontier. However, the projection identified by model (4) might not be Pareto efficient. To remedy such problem, model (5) is employed to identify the possible input excesses (s_i^-) and output shortfalls (s_r^+). If DMU *k* is not SBM-efficient, i.e., DMU *k* is inside the production possibility set spanned by DMUs excluding DMU *k*, the savings(w_i^-) and the output surpluses(w_r^+) will be zeros. Thus model (5) will function in identifying the input excesses (s_i^-) and output shortfalls (s_r^+).

It is noted that both model (4) and model (5) are fractional programming which are difficult to target global optimal solution due to their non-linear shape. However, both of them are able to be transformed into linear programming problems.

To transform model (4), let$^1/_t = 1 - (\frac{1}{s}) \sum_{r=1}^{s} w_r^+ / y_{rk}$

$$\min \delta = t + (\frac{1}{m}) \sum_{i=1}^{m} \widehat{w_i^-} / x_{ik}$$

$$s.t. tx_{ik} \geq \sum_{j=1, j \neq k}^{n} x_{ij} \widehat{\lambda_j} - \widehat{w_i^-}, i = 1, \dots, m;$$

$$ty_{rk} \leq \sum_{j=1, j \neq k}^{n} y_{rj} \widehat{\lambda_j} + \widehat{w_r^+}, r = 1, \dots, s;$$

$$1 = t - (\frac{1}{s}) \sum_{r=1}^{s} \widehat{w_r^+} / y_{rk}$$

$$\widehat{\lambda_j} \geq 0, j = 1, \dots, n, j \neq k;$$

$$\widehat{w_i^-} \geq 0, i = 1, \dots, m;$$

$$\widehat{w_r^+} \geq 0, \widehat{w_r^+} \leq ty_{rk}, r = 1, \dots, s; \qquad (6)$$

where model (6) is linear format, and $\widehat{\lambda_j} = t\lambda_j, \widehat{w_i^-} = tw_i^-, \widehat{w_r^+} = tw_r^+, i = 1, \dots, m, j = 1, \dots, n, j \neq k, r = 1, \dots, s;$

To transform model (5), let $1/_t = 1 + (\frac{1}{s}) \sum_{r=1}^{s} s_r^+ / y_{rk}$

$$\min \delta = t - (\frac{1}{m}) \sum_{i=1}^{m} \widehat{s_i^-} / x_{ik}$$

$$s.t. tx_{ik} = \sum_{j=1, j \neq k}^{n} x_{ij} \widehat{\lambda_j} - tw_i^{-*} + \widehat{s_i^-}, i = 1, \dots, m;$$

$$ty_{rk} = \sum_{j=1, j \neq k}^{n} y_{rj} \widehat{\lambda_j} + tw_r^{+*} - \widehat{s_r^+}, r = 1, \dots, s;$$

$$1 = t + (\frac{1}{s}) \sum_{r=1}^{s} \hat{s} / y_{rk}$$

$$\widehat{\lambda_j} \geq 0, j = 1, \dots, n, j \neq k;$$

$$\widehat{s_i^-} \geq 0, i = 1, \dots, m;$$

$\widehat{s_r^+} \geq 0, r = 1, \ldots, s;$ (7)

where model (7) is also linear format, and $\widehat{\lambda_j} = t\lambda_j, \widehat{s_i^-} = ts_i^-, \widehat{s_r^+} = ts_r^+, i = 1, \ldots, m, j = 1, \ldots, n, j \neq k, r = 1, \ldots, s;$

To conclude, model (6) and (7) are finalized SBM Super Efficiency DEA to be proposed for transit efficiency assessment in this dissertation.

The proposed new system takes action to assess baseline financial efficiency, target-based financial efficiency and next operational year financial efficiency for all involved bus companies.

5.3.4 Efficiency-based Target Setting

Many studies have indicated that the existing subsidy allocative process is hard to stimulate operators to improve their operation efficiency and productivity (Bergstrom, 2000; Obeng and Sakano, 2008). This is due to the fact that the bus operators are typically passive recipients of allocative decisions, rather for being actively participation into the procedures. To remedy such limitations, this study introduces an efficiency-based target setting module which allows each transit operator to set the target output and input level according to its operational constraints and capabilities for next operational year. The module is expected to enhance the relationship and mutual understanding among public transport regulators, passengers and bus operators. In this module, each bus operator is invited to specify their resource input plan, likely, cost structure plan, fleet plan, and human resource plan, etc., while the targets of output level are also needed to provide, such as volume of patronage, operational mileages and passenger satisfaction index, etc.

5.3.5 The Incentive-based Subsidy Allocation Model

5.3.5.1 Introduction

The subsidy allocation is one of the key managerial applications to support a sustainable development for public transport. It is of vital importance to allocate public resources across all involved bus companies where there is competition for resources.

Recently, the inverse DEA model has been frequently used in cost and resource allocation problems (Cook and Kress 1999; Beasley 2003; Golany, Phillips, and Rousseau 1993; Golany and Tamir 1995; Athanassopoulos 1995, 1998). To solve such a problem, the inverse DEA problem plays a role in determining the best possible inputs for given outputs such that the current efficiency value of a considered decision making unit (DMU) with respect to other DMUs remains unchanged (Saowanee et al., 2011). Specifically, in this case, one needs to consider both the competitive and cooperative situation existing among decision making units (DMUs) in addition to maintaining or improving efficiency (Du et al., 2014). Furthermore, allocating available resources (such as funds and manpower) to individual units in an appropriate manner is one of the important applications of interest. Suppose the information on input/output measures in one time period can be observed for all DMUs, decision makers desire to determine at an organizational level the most appropriate distribution of inputs resources and output targets for each DMU in the next time period. All these features are well fitted into the scope of this study where an efficiency-based target setting module has been introduced to measure the distance between actual efficiency based on observations and target-based efficiency in terms of operational and financial respectively. The obtained efficiency improvements will then transform into incentive-based allocation model to determine the level of monetary efforts. By the end of next operational year, a real change of efficiency between base year and following year will be calculated for a subsidy adjustment to the previous pre-defined subsidy plan.

5.3.5.2 The Proposed Model

This study proposed an extended inverse DEA model, which tries to answer a question, if a bus company, for instance, changes its current operational and financial efficiencies (output) into $y_{operational} + \Delta y_{operational}$, $y_{financial} + \Delta y_{financial}$, then how much cost (input) is required to achieve the goal as well as preserving the relative efficiency for selected bus company.

Using operational efficiency and financial efficiency obtained from previous steps, we firstly introduce conventional BCC DEA model, named after Banker, Charnes, and Cooper (Banker, Charnes and Cooper, 1984), to measure the relative relationship between both efficiencies and bus companies' total cost, the form of BCC model can be illustrated as follow:

$$Minimize\ \theta_0$$

$$s.t. \quad \sum_{i=1}^{n} \lambda_i x_{ji} \leq \theta_0 x_{j0}, j = 1, \dots, m$$

$$\sum_{i=1}^{n} \lambda_i y_{ki} \geq y_{k0}, k = 1, \dots, r$$

$$\sum_{i=1}^{n} \lambda_i = 1, i = 1, \dots, n$$

$$\lambda_i \geq 0, i = 1, \dots, n \qquad (1)$$

where *i=1…n, j=1…m, k=1… r,* x_{j0} is the input *j* of the considered *bus operator* ($Operator_0$), x_{ji} is the input *j* of *bus operator i*, y_{k0} is the output *k* of $operator_0$, y_{ki} is the output *k* of operator *i*, λ_i is the convex combination of operator *i*, θ_0 is the objective function or the technical efficiency value of $operator_0$. By setting financial and operational efficiency as output criteria as well as selecting total cost as input criteria, the model feedback is a relatively technical efficiency for selected operator which reveals an interaction among consumed cost, financial statements and operational conditions.

In introduced BCC model, if a considered operator changes its output (or input) values, input (or output) values of the considered operator have to be changed so as to preserve relative efficiency values. It becomes the core and working principle of inverse DEA model.

Denote the considered operator with current efficiency and cost levels by $operator_0$and the considered $operator_0$ with its efficiency and cost changes (perturbed $operator_0$) by $operator_0{}'$. In this case, the changes in financial and operational efficiencies are equal to the improvements of efficiency between base year performance and efficiency-based targets plan. The developed inverse BCC model proposed by Saowanee et al., at 2011 for a resource allocation problem is introduced to solve the problem of bus operator-level incentive-based subsidy assignment.

$Minimize\ \Delta x_0$

$$s.t. \qquad \sum_{i=1}^{n} \lambda_i x_{ji} + \lambda_{0'}(x_{j0} + \Delta x_{j0}) \leq \theta_0{}^*(x_{j0} + \Delta x_{j0}), j = 1, \dots, m$$

$$\sum_{i=1}^{n} \lambda_i y_{ki} + \lambda_{0'}(y_{k0} + \Delta y_{k0}) \leq y_{k0} + \Delta y_{k0}, k = 1, \dots, r$$

$$\sum_{i=1}^{n} \lambda_i + \lambda_{0'} = 1, i = 1, \dots, n$$

$$\lambda_i, \lambda_{0'} \geq 0, i = 1, \dots, n$$

$$x_{j0} + \Delta x_{j0} > 0 \qquad (2)$$

where $\theta_0{}^*$is the relative efficiency value of $operator_0$ before the changes in its output values calculated by model (1).

To solve the inverse BCC model for the subsidy allocation problem, we need to find the value of $\Delta x_0 = (\Delta x_{10,} \Delta x_{20}, \dots, \Delta x_{m0})$, which keeps the relative efficiency values of all bus operators unchanged. This can be done by solving model (2). However, this model is in the form of nonlinear programming, which is hard to target a global optimal solution. To remedy such a problem, Saowanee et al., (2011) propose and further prove its feasibility of a multi-objective linear programming model, which gives an optimal solution for the inverse BCC model.

$$Minimize\, W^T\, \Delta x_0$$

$$s.t. \qquad \sum_{i=1}^{n} \lambda_i x_{ji} \le \theta_0{}^*(x_{j0} + \Delta x_{j0}), j = 1, \dots, m$$

$$\sum_{i=1}^{n} \lambda_i y_{ki} \ge y_{k0} + \Delta y_{k0}, k = 1, \dots, r$$

$$\sum_{i=1}^{n} \lambda_i = 1, i = 1, \dots, n$$

$$\lambda_i \ge 0, i = 1, \dots, n$$

$$x_{j0} + \Delta x_{j0} > 0$$

$$W^T \in R^m \tag{3}$$

The model has been proved that it is able to find a Pareto solution under any positive vector, $W^T \in R^m$.

The proposed inverse model is considered for a subsidy allocation problem, where increases of some outputs and decreases of the other outputs for the considered transit operators can be processed simultaneously. The output of the proposed model is an optimized minimumΔx_0 , which is the total cost change by the effect of improvements in efficiency. It is further regarded as the cost-orientated effort to achieve targets and improve companies' performances.

The module of incentive-based subsidy allocation serves in pre-determining the amount of subsidy to each company depending on target-based efficiency improvements, and also in distributing subsidies referring to the actual changes between base year and following year. The generated two subsidy allocation plans further moves together to implement an adjustment procedure which will be described in the next section.

5.3.6 The Incentive-based Subsidy Allocation Adjustment

The introduction of efficiency-based target setting allows each operator to define its efficiency improvement plan which is further used to pre-determine the level of subsidy by proposed incentive-based subsidy model. In order to receive full target-based grant, all involved companies are encouraged to meet their goals. However, by the end of next operational year, a procedure of subsidy adjustment is still necessary to be delivered which is for checking and monitoring whether the pre-set targets are achieved or not. The adjustment is implemented according to the collected operational and financial data of next operational year. There are three cases need to be clarified:

- A full grant is assigned when the considered bus company completes its targets;
- A corrected subsidy plan is assigned and proportional to the percentage complete when the considered bus company does not entirely meet its targets;
- In addition to receive a full grant, an extra credit is also awarded into next operational year when the performance of considered bus company beyond its expectation.

5.3.7 Case Study

5.3.7.1 Introduction

To illustrate the applicability of the proposed framework and models in allocating incentive-based subsidy at operator level, this section has selected 5 bus companies in Chongqing Metropolitan as case study.

As a junction of Southwestern and central China as well as being a one of the largest cities upstream on the Yangtze River, the city is one of the most important transportation hubs as it connects eastern and western China with an area of 82,400 square kilometers (31,800 square miles) and 30 million population. The city's transportation system has been very well developed in recent years and more than 8000 buses managed by 5 bus companies are offering services and 500 bus routes and 6 metro lines were in operation by end of 2014.

In case study, year 2014 is selected as the base year which the collected original data is further categorized into financial subset and operational subset. The proposed models are then practiced to evaluate base year financial and operational performances for all five companies according to two data subsets respectively. With obtained efficiency scores, two scenarios, basic and proactive plans, are designed to separately gather the efficiency improvements at financial and operational perspectives. Subsequently, the introduced incentive-based subsidy allocation model is adopted to determine the amount of subsidy depending on each company' improvement.

5.3.7.2 Base Year Transit Operator Financial and Operational Efficiency Assessment

Data of 2014 used in this study were collected from the annual report of Chongqing Bus Group. Specifically, performances of five bus companies are reflected via 12 different indictors, namely, size of bus fleet, number of employee, consumed fuel, labor cost, fuel cost, depreciation expense, repair and maintenance cost, mileage, number of patronage, profit, number of runs, passenger satisfaction index. To further distinguish operators' financial and operational performance, all collected indictors are classified into two different subsets. The financial subset contains labor cost, fuel cost, depreciation expense, repair and maintenance cost, mileage, number of patronage, profit, number of runs, passenger satisfaction index, while the operational subset includes size of bus fleet, number of employee, consumed fuel, mileage, number of patronage, profit, number of runs, passenger satisfaction index. Table 5.1 records the data of 2014.

Table 5.1 2014 Original data

Bus Operator	Fleet	Employee	Gas (m3/100k)	Labor Cost (10^4Yuan)	Fuel Cost (10^4Yuan)	Depreciation expense (10^4Yuan)	Repair Cost (10^4Yuan)	Mileage (10^4Km)	Patronage (10^4Trips)	Profit (Yuan/100km)	No.Runs	Satisfaction Index
Yiqi	1191	3562	41.88	31362	11985	6531	3158	8554	27796	522	526.38	94.02
Xibu	2736	8167	40.65	75290	26276	13812	7748	19859	63038	527	1168.02	91.88
Third	1233	3735	42.17	26529	11884	5750	2990	8033	25014	515	493.21	93.12
Fifth	923	2761	32.89	20686	7333	5691	2244	6646	13700	437	354.90	92.56
Liangjiang	2544	7608	43.49	57888	25336	14126	7524	17392	56464	546	902.86	95.23

Table 5.2 2014 Chongqing Bus Company financial efficiency

Bus Operator	Labor Cost (10^4Yuan)	Fuel Cost (10^4Yuan)	Depreciation expense (10^4Yuan)	Repair Cost (10^4Yuan)	Mileage (10^4Km)	Patronage (10^4Trips)	Profit (Yuan/100Km)	No.Runs	Satisfaction Index	Financial Efficiency
Yiqi	31362	11985	6531	3158	8554	27796	522	526.38	94.02	1.0346
Xibu	75290	26276	13812	7748	19859	63038	527	1168.02	91.88	1.0377
Third	26529	11884	5750	2990	8033	25014	515	493.21	93.12	1.0631
Fifth	20686	7333	5691	2244	6646	13700	437	354.90	92.56	1.1036
Liangjiang	57888	25336	14126	7524	17392	56464	546	902.86	95.23	1.0233

Table 5.3 2014 Chongqing Bus Company operational efficiency

Bus Operator	Fleet	Employee	Gas (m3/100km)	Mileage (10^4Km)	Patronage (10^4Trips)	Profit (Yuan/100Km)	No.Runs	Satisfaction Index	Opertional Efficiency
Yiqi	1191	3562	41.88	8554	27796	522	526.38	94.02	0.7127
Xibu	2736	8167	40.65	19859	63038	527	1168.02	91.88	0.9569
Third	1233	3735	42.17	8033	25014	515	493.21	93.12	0.6039
Fifth	923	2761	32.89	6646	13700	437	354.90	92.56	0.8220
Liangjiang	2544	7608	43.49	17392	56464	546	902.86	95.23	0.6048

And then, the cone-based enhanced DEA model for operational efficiency assessment at operator level is used to evaluate bus companies' operational performance, where size of bus fleet, number of employee and consumed fuel are input indicators while mileage, number of patronage, profit, number of runs and passenger satisfaction index are output criteria. During the process, the proposed Fuzzy-AHP model is firstly activated to generate two constraint cones for input and output group indicators, as following:

$$Input\ group\ cone = \begin{bmatrix} 1.0000 & 1.0263 & 6.2092 \\ 0.9744 & 1.0000 & 6.1622 \\ 0.1611 & 0.1623 & 1.0000 \end{bmatrix}$$

$$Output\ group\ cone = \begin{bmatrix} 1.0000 & 0.4070 & 6.4779 & 1.2447 & 7.5331 \\ 2.4570 & 1.0000 & 7.9766 & 2.7174 & 9.0000 \\ 0.1544 & 0.1254 & 1.0000 & 0.1609 & 2.0634 \\ 0.8034 & 0.3680 & 6.2133 & 1.0000 & 7.2934 \\ 0.1327 & 0.1111 & 0.4846 & 0.1371 & 1.0000 \end{bmatrix}$$

Two cone matrixes which contain preference information over indicators have been integrated into DEA model. Later on, the enhanced model functions to assess the operational efficiency for bus companies (see Table 5.2 for results). The obtained scores clearly reveal that Xibu Bus Company performed better than others while Third Bus Company ranked last due to an inefficient usage of input resources. Reasonably, as the largest company, Xibu owns the biggest size of bus fleet and employee while it also produces the highest number of runs and passengers; it is not surprising that Xibu stays ahead of others.

In following step, the proposed SBM Super Efficiency DEA model introduced from **Section 5.3.3** is practiced to assess operators' financial efficiency based on financial dataset where labor cost, fuel cost, depreciation expense, repair and maintenance cost are considered as input group while mileage, number of patronage, profit, number of runs and passenger satisfaction index are treated as output criteria where Table 5.3 descripts the result. After an implementation of two stages revised SBM Super Efficiency DEA models, the financial performances of five companies have been clearly discriminated where the Fifth Bus Company ranks top while Liangjiang Bus Company lags behind others in 2014. To explore the reasons, small firms always have a moderately tight financial and budget control policy which leads them to a high financial efficiency. Consequently, as the smallest enterprise in five bus companies, the Fifith Bus Company has obviously delivered a good financial report compared with other four. It is noted that the financial efficiencies of all five companies are excessed 1 which indicates an effective usage of funding resources.

Figure 5.3and 5.4 accommodates and compares both operational and financial efficiency for all five companies.

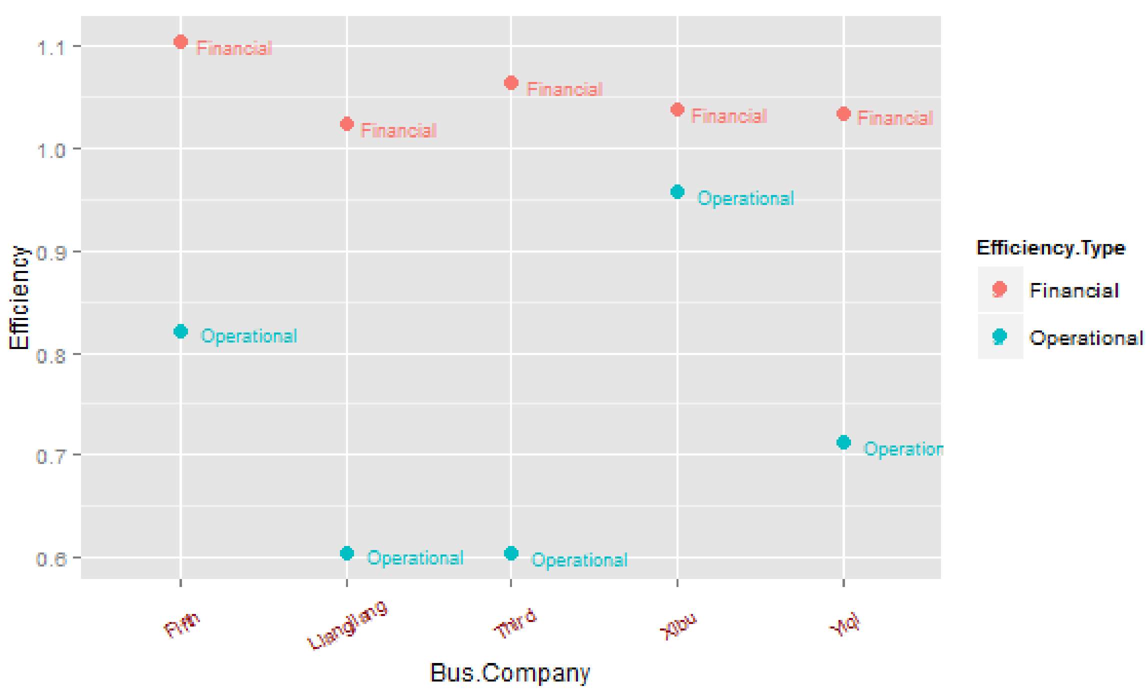


Figure 5.3 Operational and financial efficiencies of five companies

Figure 5.4 Operational and financial efficiencies of five companies

By analyzing from a general prospective, the Fifth and Xibu Bus companies performed better than others from both financial and operational angles while Liangjiang and Third Bus companies have a manifest difference for their financial and operational performance in 2014.

5.3.7.3 Scenarios Design of Efficiency-based Target Setting

In this section, two scenarios of target setting are created for subsidy pre-determination of year 2015. The general principle of constructing scenarios is to take aims at improving their efficiencies which generally require a reduction in resource input as well as increasing output level. Two hypothetical scenarios are designed to gain both operational and financial improvements compared to base year situation. The first one is called “Basic Plan” while another one is “Proactive Plan”. The main difference between two scenarios is the degree of change on input and output criteria. In “Basic Plan”, the bus operators are encouraged to slightly raise their output on items of mileage, patronage, profit, number of runs and passenger satisfaction index, in the meantime, they are also required to reduce cost and gas consumption. However, the number of buses is permitted to be raised, which is for achieving the targets of increasing passenger volume and number of runs. By review of “Proactive Plan”, by compared with “Basic Plan”, all five bus companies are certainly requested to increase more on their output group as well as decreasing more on their input resource. Differently, the number of buses is set to decline which could be an aggressive objective in "Proactive Plan". The detailed changes of "Basic Plan" and "Proactive Plan" are demonstrated in Table 5.4 and Table 5.5 respectively. In addition, two figures, Figure 5.5 and Figure 5.6, have been created to better illustrate plans.

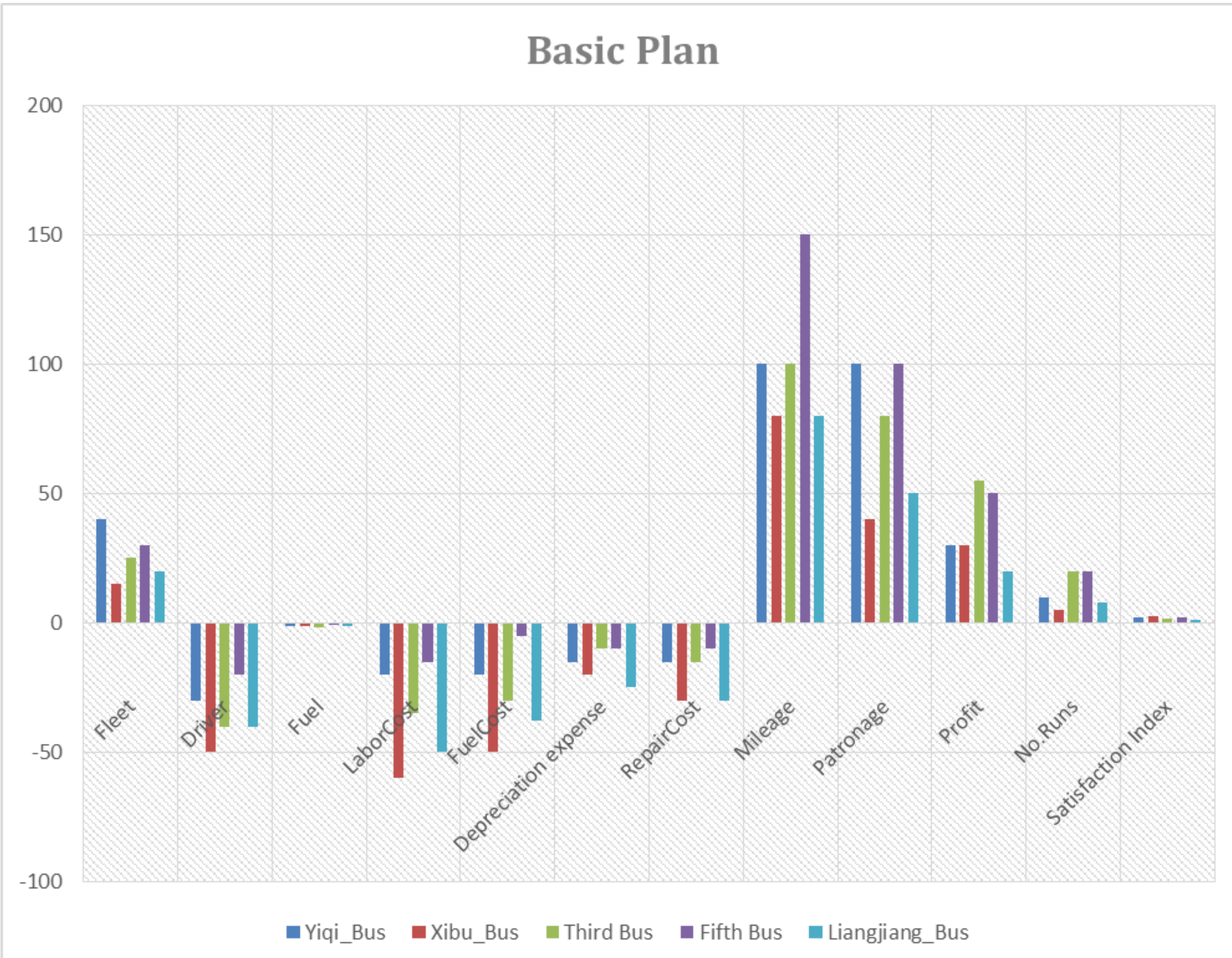


Figure 5.5 Basic plan

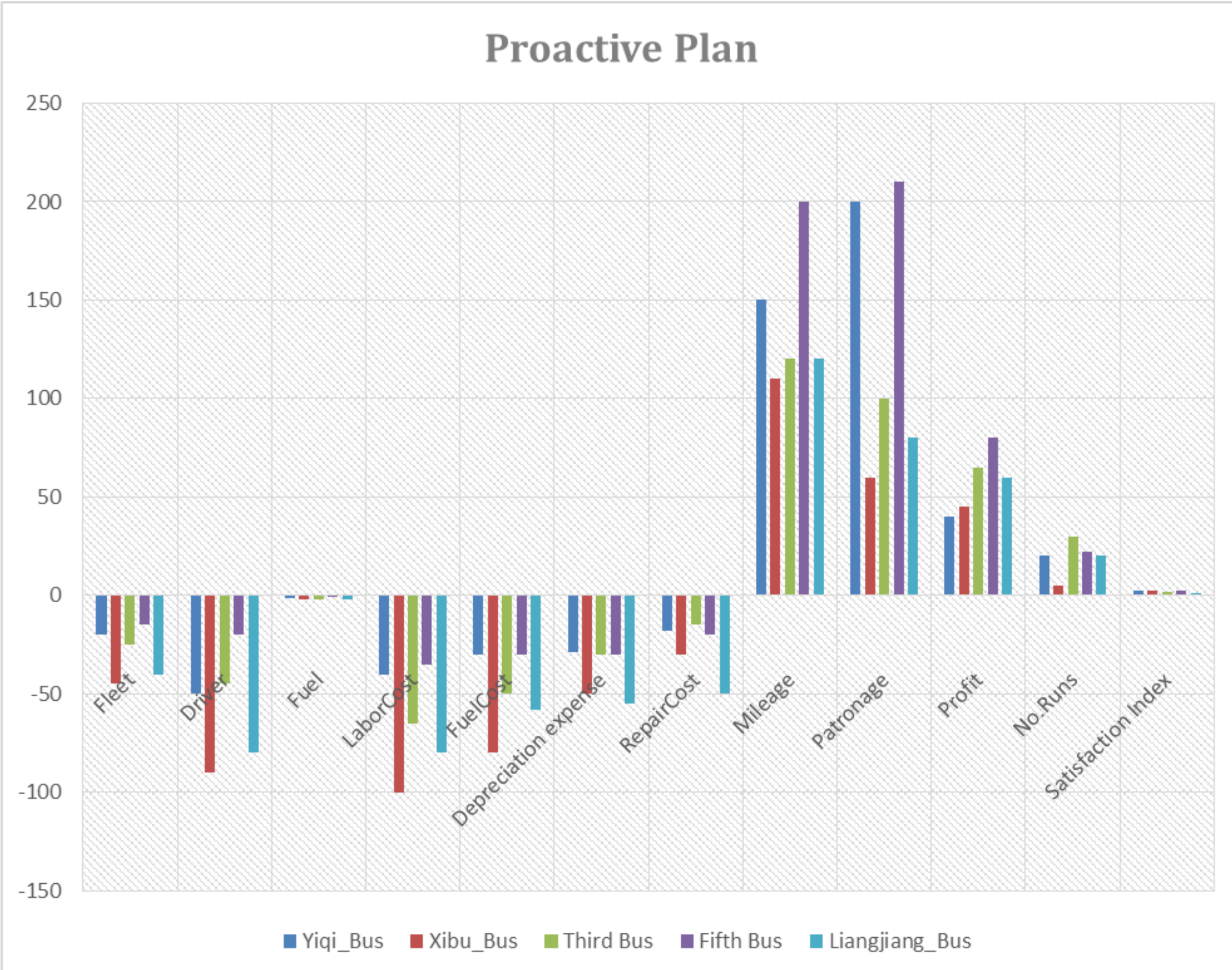


Figure 5.6 Proactive plan

5.3.7.4 Target-based Operational and Financial Efficiency Assessment

Similar to the procedure of base year efficiency assessment, the whole data set of two designed scenarios are further classified into operational subset and financial subset. With using corresponding data subsets, the proposed cone-based enhanced DEA model and revised SBM Super Efficiency model are activated to evaluate bus companies' target-based operational and financial performance respectively.

Table 5.4 Basic plan

Bus Operator	Fleet	Employee	Gas (m3/100km)	Labor Cost (10^4Yuan)	Fuel Cost (10^4Yuan)	Depreciation expense (10^4Yuan)	Repair Cost (10^4Yuan)	Mileage (10^4Km)	Patronage (10^4Trips)	Profit (Yuan/100Km)	No.Runs	Satisfaction Index
Yiqi	+40	-30	-1.2	-20	-20	-15	-15	+100	+100	+30	+10.00	+2
Xibu	+15	-50	-1.1	-60	-50	-20	-30	+80	+40	+30	+5.00	+2.5
Third	+25	-40	-1.8	-35	-30	-10	-15	+100	+80	+55	+20.00	+1.8
Fifth	+30	-20	-0.8	-15	-5	-10	-10	+150	+100	+50	+20.00	+2.3
Liangjiang	+20	-40	-1.3	-50	-38	-25	-30	+80	+50	+20	+8.00	+1

Table 5.5 Proactive plan

Bus Operator	Fleet	Employee	Gas (m3/100km)	Labor Cost (10^4Yuan)	Fuel Cost (10^4Yuan)	Depreciation expense (10^4Yuan)	Repair Cost (10^4Yuan)	Mileage (10^4Km)	Patronage (10^4Trips)	Profit (Yuan/100Km)	No.Runs	Satisfaction Index
Yiqi	-20	-50	-1.5	-40	-30	-29	-18	+150	+200	+40	+20.00	+2.3
Xibu	-45	-90	-1.8	-100	-80	-50	-30	+110	+60	+45	+5.00	+2.5
Third	-25	-45	-1.9	-65	-50	-30	-15	+120	+100	+65	+30.00	+1.8
Fifth	-15	-20	-1.1	-35	-30	-30	-20	+200	+210	+80	+22.00	+2.5
Liangjiang	-40	-80	-2.1	-80	-58	-55	-50	+120	+80	+60	+20.00	+1

The target-based efficiencies and the corresponding improvements are stored in Table 5.6 and Table 5.7 which are for basic plan and proactive plan respectively.

Table 5.6 "Basic Plan" efficiency

Bus Company	Base Year		Basic Plan		Improvement	
	Financial	Operational	Financial	Operational	Financial	Operational
Yiqi	1.03460	0.71270	1.04042	0.74521	0.00582	0.03251
Xibu	1.03770	0.95690	1.04059	0.97544	0.00289	0.01854
Third	1.06310	0.60390	1.13155	0.69527	0.06845	0.09137
Fifth	1.10360	0.82200	1.10913	0.84188	0.00553	0.01988
Liangjiang	1.02330	0.60480	1.14050	0.61409	0.11720	0.00929

Table 5.7 "Proactive Plan" efficiency

Bus Company	Base Year		Proactive Plan		Improvement	
	Financial	Operational	Financial	Operational	Financial	Operational
Yiqi	1.03460	0.71270	1.04546	0.78723	0.01086	0.07453
Xibu	1.03770	0.95690	1.04347	0.99105	0.00577	0.03415
Third	1.06310	0.60390	1.14300	0.69999	0.07990	0.09609
Fifth	1.10360	0.82200	1.10987	0.86059	0.00627	0.03859
Liangjiang	1.02330	0.60480	1.19900	0.61672	0.17570	0.01192

The two tables tell that both of scenarios have successfully improved both financial and operational efficiencies by reason of the optimization on input resources and output targets. In terms of financial performance, as the last in base year, Liangjiang Bus Company makes a greatest progress in both plans by compared with other four companies, specifically, 0.11720 and 0.17570 correspondingly. By review of operational prospective, Third Bus Company produces two significant increases from 0.60390 to 0.69527, and from 0.60390 to 0.69999 at basic and proactive plan respectively. To compare two scenarios, the efficiency scores of "Proactive Plan" certainly raise more than "Basic Plan" due to the more significant changes on both input and output criteria. Two sets of pictures are used to demonstrate the differences and general trends in two plans in a more straightforward approach.

Figure 5.7 Efficiency change of basic plan

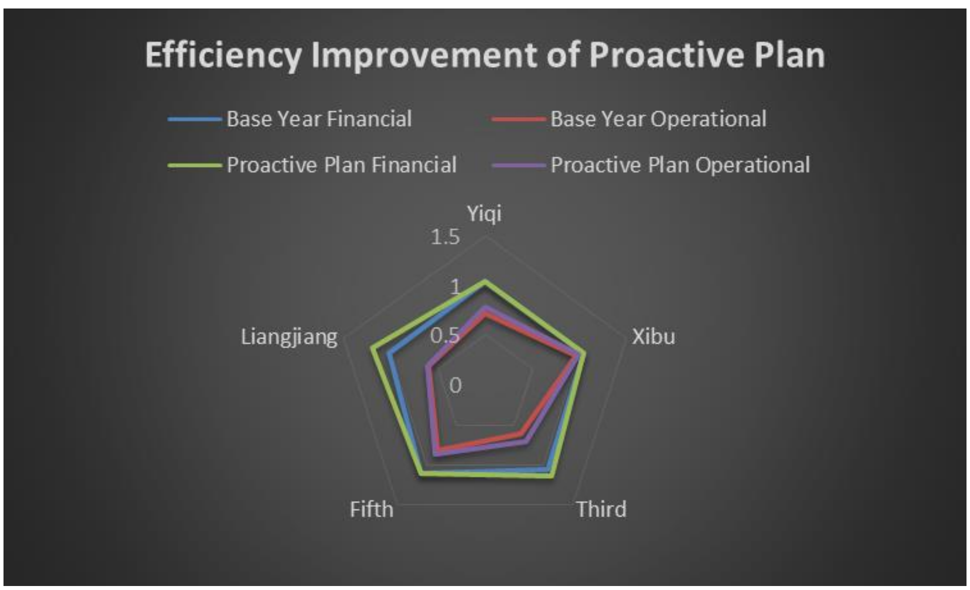


Figure 5.8 Efficiency change of proactive plan

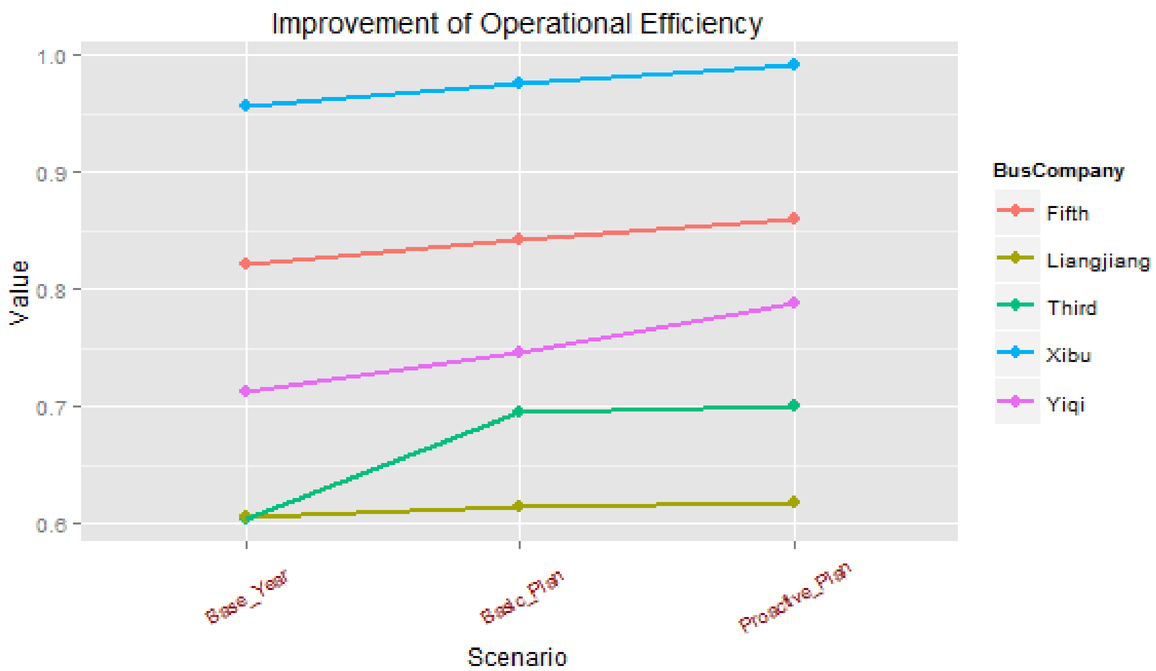


Figure 5.9 Improvement of operational efficiency

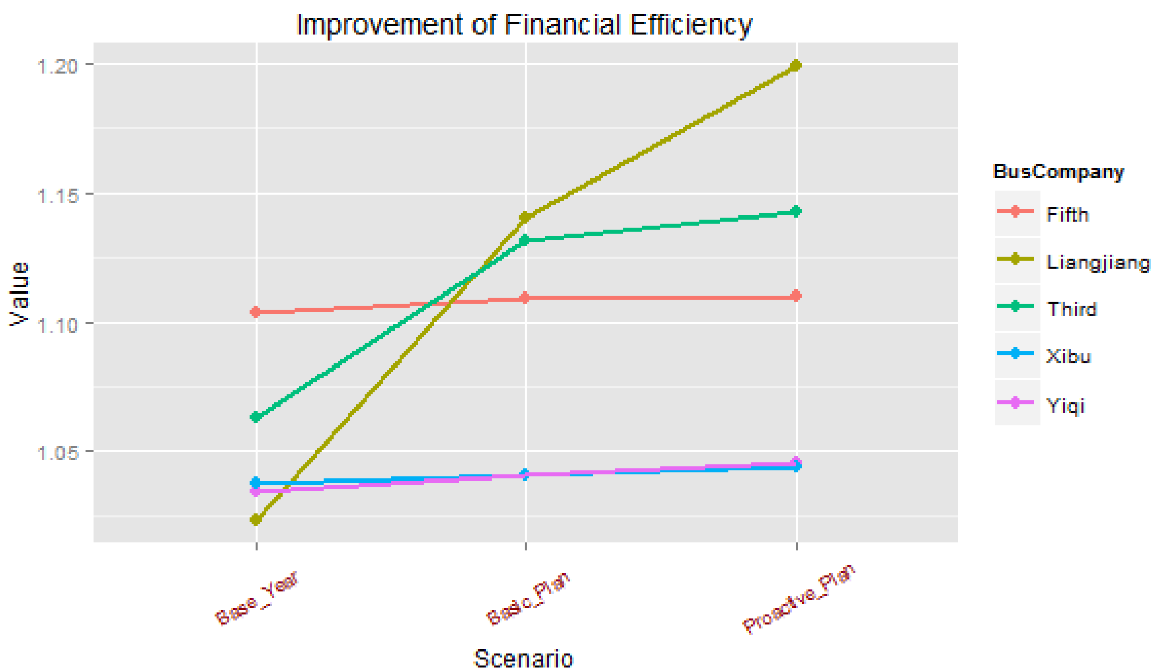


Figure 5.10 Improvement of financial efficiency

5.3.7.5 Target-based Incentive-based Subsidy Allocation

The previous section helps each bus operator gain their efficiency improvements based on target setting. Those obtained variations turn to be defined as Δy which are further used to determine the correspondingΔx to preserve the relative efficiency relationship between input cost and financial/operational efficiency of base year by proposed incentive-based subsidy allocation model. The produced Δx is then considered as the monetary effort to achieve the targets and improve both operational and financial efficiencies.

First of all, the introduced BCC model is processed to measure base year relative efficiency between operational/financial efficiencies and bus companies' total cost. It reveals a relationship between cost and efficiency which is suggested to be steady and preserved due to a mutual market and a sustainable financial status. The input cost here is recommended to convert into unit cost (cost/per km) to avoid an uncontrolled impact caused by the shape difference in business scale. Take Chongqing as an example, the business scale of Xibu Bus is almost triple the size of Fifth Bus which is reflected by the difference in their cost structure, operated mileage, patronage and etc. Next, the revised inverse DEA is operated to determine the change of cost corresponding to the efficiency improvements. The optimized value is regarded as the cost-orientated effort to achieve the objectives. Table 5.8 and 5.9 show the results in accordance with two designed scenarios.

Table 5.8 Subsidy allocation plan of basic scenario

Bus Company	Base Year Unit Cost(Yuan/km)	Improvement of Basic Plan		BCC Efficiency	Subsidy (Yuan/km)	Mileage (10^4Km)	Total (Yuan)
		Financial	Operational				
Yiqi	6.20	0.00582	0.03251	0.9210	0.04792	8554	4099180.30
Xibu	6.20	0.00289	0.01854	1.0000	0.41158	19859	81734838.12
Third	5.87	0.06845	0.09137	0.9727	0.14902	8033	11970873.00
Fifth	5.71	0.00553	0.01988	1.0000	0.62693	6646	41665688.05
Liangjiang	6.03	0.11720	0.00929	0.9551	0.14864	17392	25850703.55

Table 5.9 Subsidy allocation plan of proactive scenario

Bus Company	Base Year Unit Cost(Yuan/km)	Improvement of Proactive Plan		BCC Efficiency	Subsidy (Yuan/km)	Mileage (10^4Km)	Total (Yuan)
		Financial	Operational				
Yiqi	6.20	0.01086	0.07453	0.9210	0.189889	8554	16243130.72
Xibu	6.20	0.00577	0.03415	1.0000	0.491567	19859	97620230.95
Third	5.87	0.07990	0.09609	0.9727	0.209694	8033	16844735.09
Fifth	5.71	0.00627	0.03859	1.0000	0.654187	6646	43477241.44
Liangjiang	6.03	0.17570	0.01192	0.9551	0.465417	17392	80945324.64

Figure 5.11 and 5.12 record the relationship between subsidy allocation plans and efficiency improvements.

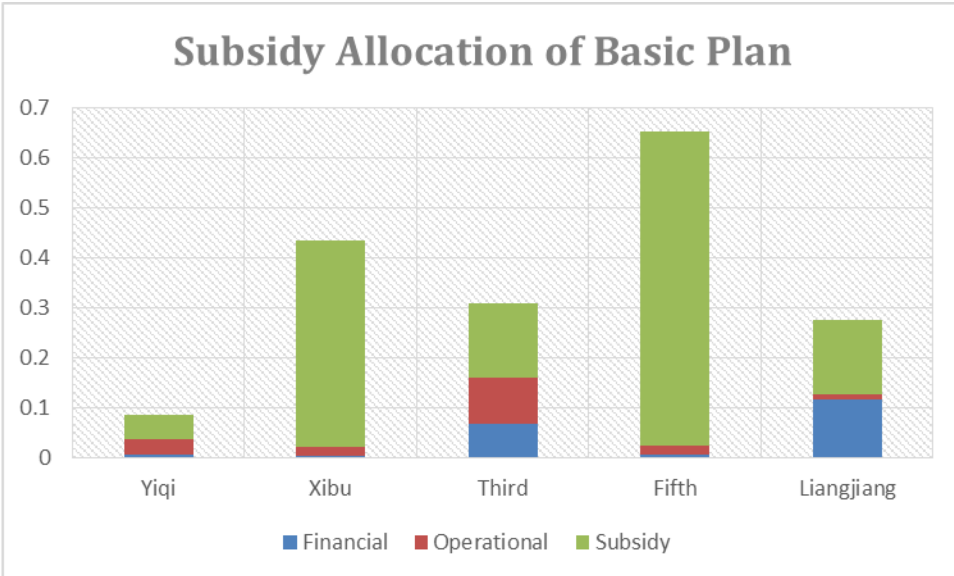


Figure 5.3 Subsidy allocation of basic plan

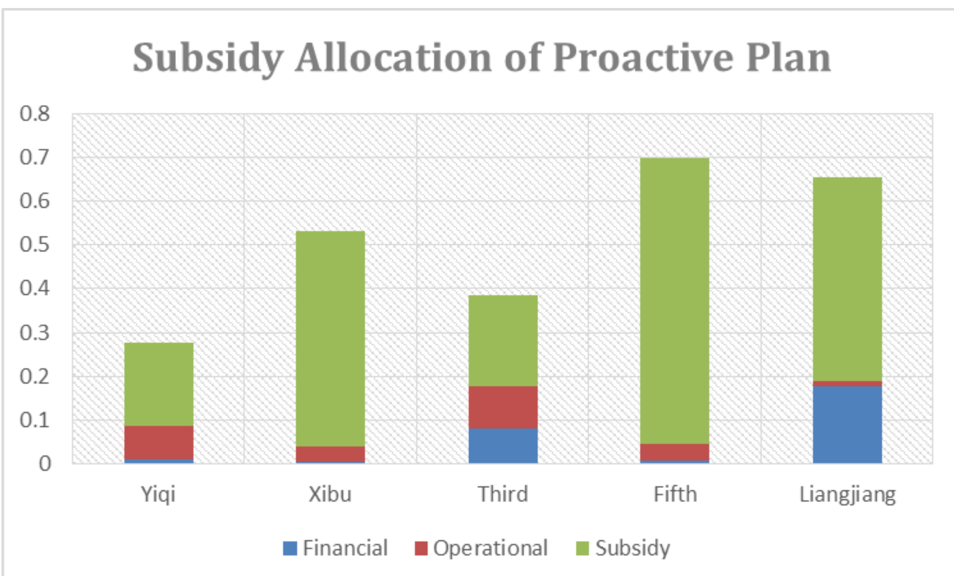


Figure 5.14 Subsidy allocation of proactive plan

From tables and figures, it is observed that level of allocated subsidy is not purely subject to the proportion of efficiency increase, furthermore, it also refers to the relative efficiency between cost and base year financial/operational efficiency. The efficient units (BCC efficiency equal to 1) typically require more subsidies to maintain their outstanding performance, and the more increments of efficiencies certainly lead to a higher level of subsidy. Also, the “Proactive Plan” announces more subsidy than “Basic Plan” to achieve a more ambitious strategy.

5.3.7.6 Subsidy Allocation Adjustment

By the end of 2015, a procedure of subsidy adjustment is suggested to process according to the following principles:

- A full grant is assigned when the considered bus company completes its targets;
- A corrected subsidy plan is assigned and proportional to the percentage complete when the considered bus company does not entirely meet its targets;
- In addition to receive a full grant, an extra credit is also awarded into next operational year when the performance of considered bus company beyond its expectation.

After a complete process of incentive-based subsidy allocation, the assigned funding exerts positive influences and efforts in improving both operational and financial efficiencies of bus operators which could further help public transport sustainable development. Unlike traditional cost-based subsidy allocation strategy, the proposed mechanism stimulates Bus Company to actively be involved in whole process, rather for being a passive responder.

5.3.8 Conclusion

This section develops an incentive-based subsidy allocation mechanisms and the corresponding quantitative approach that can allocate limited subsidies to urban transit operators according to their operational and financial efficiencies. A comprehensive evaluation and decision framework is developed, consisting of key modules of baseline assessment of transit operational and financial efficiency, efficiency-based target setting and pre-evaluation, incentive-based subsidy allocation, as well as feedback and subsidy adjustment.

When evaluating the operational efficiency of transit operators, a Constrained Cone-based Enhanced DEA developed in **Section 4.3** is activated. Introduction of preference cone constraints into the DEA model is critical for decision makers to incorporate their preferences or important policies over inputs/outputs into the performance evaluation and subsidy allocation process. When evaluating the financial efficiency of transit operators, a revised SBM Super Efficiency Model is developed to directly account for input and output slack in efficiency measurements, with the advantage of capturing the whole aspect of inefficiency.

Using the above obtained financial and operational efficiencies as the new set of outputs and the operational cost as the new set of inputs, an extended inverse DEA model is developed to allocate incentive-based subsidy. The model allows each transit operator to set the target output and efficiency level according to its operational constraints and capabilities. The corresponding improvements are then processed by the inverse DEA model to identify extra inputs to each transit operator (i.e. the subsidy allocated). Therefore, allocation of the subsidy mainly depends on the improvement of a transit operator's performance rather than the running cost.

The proposed model has selected Chongqing Municipality, China as a case study for integrated performance evaluation and subsidy allocation of five transit operators regulated by the municipal government. Results demonstrate an advantage of proposed strategy over traditional framework.

5.4. Bus Route-level Incentive-based Subsidy Allocation model

5.4.1 Research Motivation

Section 5.3 has targeted the goals and its corresponding incentive-based subsidy for each bus company. In this section, focus has shifted from bus operators to bus routes, specifically, how to apportion those operator-level subsidies and to prioritize them into routes determines the company's overall operational efficiency. Some key issues may include:

- Additional input and output assignment: decision maker desires (or is obliged) to allocate additional subsidy resource to the inputs of the routes and to define a reasonable target for the output-level of the involved routes;
- Fixed resource allocation issue: the sum of assigned subsidies for each bus route should equal the amount of subsidy for their respective bus company;
- Target setting issue: the sum of set targets for each bus routes should equal the amount of targets for their respective bus company;
- Centralized resource allocation issue: there are situations in which all the routes fall under the umbrella of a centralized bus company that oversees them. In another word, all of the units belong to the same organization (public or private), which provides the units with the necessary resources to obtain their outputs.

In order to include all features in, this section develops a new system to allocate fixed subsidy and set target to bus routes simultaneously. The model functions to subdivide subsidies and targets from bus company level to bus route level, and further to optimize resource configuration for all selected routes.

5.4.2 The Proposed Model

5.4.2.1 The Proposed Multi-objective Fractional Programming-based DEA Model

In the direction of reflecting the competitions for limited subsidy among all bus routes and further allocating company's subsidy and targets to involved routes simultaneously, this section develops a multi-objective fractional programming (MOFP)-based DEA model. MOFP is defined as a specific type of multi-objective optimization (MOP), which is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints. In addition to basic concept of MOP, MOFP features in executing fractional maximization subject to a set of constraints in order to tackle such complex and ill-structured decision problems.

It assumes that there are *n* DMUs under consideration with *m* inputs and *r* outputs. The following MOFP problem can be used to maximize the efficiency score of all DMUs simultaneously:

$$max\ W = \left\{\frac{\sum_{r=1}^{s} u_r y_{r1}}{\sum_{i=1}^{m} v_i x_{i1}}, \frac{\sum_{r=1}^{s} u_r y_{r2}}{\sum_{i=1}^{m} v_i x_{i2}}, \dots, \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}\right\}$$

$$s.t. \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \forall j,$$

$$u_r, v_i \geq \varepsilon, \forall r, i. \qquad (1)$$

Many studies have been developed to solve the MOFP problem. Goal programming, as one of the seminal methods for multi-objective optimization (Tamiz and Romero, 1998), is required to set aspiration levels for the objective functions. Then, deviations from these aspiration levels are minimized as a preferred solution. An objective function jointly with an aspiration level is referred to as a goal. Based on the concept of GP method, model (1) can be converted into the following non-linear model for identifying a set of common weights (Davoodi and Zhiani, 2012):

$$\min \sum_{j=1}^{n} (\varphi_j^- + \varphi_j^+)$$

$$s.t. \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} + \varphi_j^- - \varphi_j^+ = A_j, \forall j,$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \forall j,$$

$$\varphi_j^-, \varphi_j^+ \geq 0, u_r, v_i \geq \varepsilon, \forall r, i, j. \qquad (2)$$

where A_j, *j = 1,…,n,* represents the goal of the *jth* objective function, $\varphi_j^- \, and \varphi_j^+$ are the under-achievement (so-called negative deviation) and over-achievement (so-called positive deviation) of the *jth* goal, respectively. A_j is set to unity in model (2) since in the conventional DEA models, each DMU desires to maximize the efficiency score.

Lotf., et al., (2013) has simplified and linearized the solving model by eliminating redundant constraints and substitution variables $\varphi_j^- + \varphi_j^+$ with φ_j, and then the modified linear programming is illustrated as following:

$$\min \sum_{j=1}^{n} \varphi_j$$

$$s.t. \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \varphi_j = 0, \forall j,$$

$$\varphi_j \geq 0, u_r, v_i \geq \varepsilon, \forall r, i, j. \qquad (3)$$

By solving above model, a set of optimal value, $(u_r{}^*, v_i{}^*, \varphi_j{}^*)$, is obtained to further calculate the efficiency scores of DMU_j, *j=1,…,n,* as follows:

$$\theta_j^* = \frac{\sum_{r=1}^{s} u_r{}^* y_{rj}}{\sum_{i=1}^{m} v_i{}^* x_{ij}} = 1 - \frac{\varphi_j{}^*}{\sum_{i=1}^{m} v_i x_{ij}}, \forall j.$$

5.4.2.2 The Proposed Route-level Subsidy Allocation and Target Setting Model

The proposed MOFP-based DEA model provides a potential of adding input and output to all DMUs simultaneously. If we consider subsidy as a new additional input resource, in the meantime, the various types of targets are treated as new outputs. A new model for allocating fixed subsidy resource as well as setting targets to all involved bus routes synchronously are proposed by referring to the revised MOFP-based DEA model developed by Lotf., et al., in 2013.

Let us consider a bus company consisting of *n* independent bus routes under the evaluation process that each $route_j$, $j = 1, \dots, n$, use *m* inputs, $x_{ij} \in R^+, (i = 1, \dots, m; j = 1, \dots, n)$ to produce *s* outputs, $y_{rj} \in R^+, (r = 1, \dots, s; j = 1, \dots, n)$. In this centralized system, Bus Company has received incentive-based subsidy, F, and it wants to allocate this funding to each route. Accordingly, the company expects to achieve *p* fixed outputs, $G_w \in R^+, w = 1, \dots, p$, as targets set for each bus routes. Noticeably, G_w could be the targets from plan designed in Section 5.3.3 or could also be the new targets set by companies. The non-negative variables $\overline{f_J}$ and $\overline{g_{wJ}}$ present the allocated subsidy and allocated targets to $route_j$, respectively. Thus, the relations $\sum_{j=1}^{n} \overline{f_J} = F \; and \; \sum_{j=1}^{n} \overline{g_{wJ}} = G_w, w = 1, \dots p$, must be held. Hence, the following system can be developed:

$$\frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{w=1}^{p} u_{s+w} \overline{g_{wJ}}}{\sum_{i=1}^{m} v_i x_{ij} + v_{m+1} \overline{f_J}} = 1, \forall j,$$

$$\sum_{j=1}^{n} \overline{f_J} = F$$

$$\sum_{j=1}^{n} \overline{g_{wJ}} = G_w, \forall w,$$

$$u_r, v_i, u_{s+w}, v_{m+1} \geq \varepsilon, \overline{f_J}, \overline{g_{wJ}} \geq 0, \forall r, i, k, w, j. \qquad (4)$$

By review of above system, the first constrain in model (4) is able to guarantees that each bus route would be assigned an appropriate level of subsidy and target which are further utilized to optimize input and output resource configuration to reach efficient status. In other words, each bus route is able to be allocated its best portion and maximized their utilization. In the meantime, the bus operator-level subsidy and targets could successfully distribute into all involved routes.

For sake of linearizing model (4), two sets of alteration variables are firstly introduced:

$$u_{s+w}\overline{g_{wj}} = g_{wj}$$

$$v_{m+1}\overline{f_j} = f_j$$

So the model (4) can transform into:

$$\frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{w=1}^{p} g_{wj}}{\sum_{i=1}^{m} v_i x_{ij} + f_j} = 1, \forall j,$$

$$\sum_{j=1}^{n} f_j \quad = v_{m+1}F,$$

$$\sum_{j=1}^{n} g_{wj} = \ u_{s+w}G_w, \forall w,$$

$$u_r, v_i, u_{s+w}, v_{m+1} \geq \varepsilon, f_j, g_{wj} \geq 0, \forall r, i, k, w, j. \qquad (5)$$

In the second step, two multipliers λ_j *and* μ_j are assigned to all additional subsidy input and all additional outputs by reason of reflecting the effects of the present input and output values in allocating subsidy and setting output targets for a certain routes. λ_j and μ_j are given by:

$$\lambda_j = (\frac{1}{m})\sum_{i=1}^{m}[x_{ij}/\sum_{t=1}^{n} x_{it}]$$

$$\mu_j = (\frac{1}{s})\sum_{i=1}^{s}[y_{rj}/\sum_{t=1}^{n} y_{rt}]$$

$$\sum_{j=1}^{n}\lambda_j = \sum_{j=1}^{n}\mu_j = 1$$

Then, several extra variables are defined for linear programming based on GP concept, in detail, we use the negative and positive deviational variables for $f_j$and g_{wj} denoted by$\left(\alpha_j^-, \alpha_j^+\right)$ and $\left(\beta_{wj}^-, \beta_{wj}^+\right)$. Model (5) is now converted into:

$$\min \sum_{j=1}^{n}((\alpha_j^- + \alpha_j^+) + \sum_{w=1}^{p}(\beta_{wj}^- + \beta_{wj}^+))$$

$$s.t. \frac{\sum_{r=1}^{s} u_r y_{rj} + \sum_{w=1}^{p} g_{wj}}{\sum_{i=1}^{m} v_i x_{ij} + f_j} = 1, \forall j,$$

$$f_j + \alpha_j^- - \alpha_j^+ = v_{m+1}\lambda_j F, \forall j,$$

$$g_{wj} + \beta_{wj}^- - \beta_{wj}^+ = u_{s+w}\mu_j G_w, \forall j, w,$$

$$\sum_{j=1}^{n} f_j \;\; = v_{m+1}F,$$

$$\sum_{j=1}^{n} g_{wj} = \; u_{s+w}G_w, \forall w,$$

$$u_r, v_i, u_{s+w}, v_{m+1} \geq \varepsilon, f_j, g_{wj}, \alpha_j^-, \alpha_j^+, \beta_{wj}^-, \beta_{wj}^+ \geq 0, \forall r, i, k, w, j. \tag{6}$$

Finally, after implement cross-multiplication method, the fractional programming model (6) can be transformed into linear programming problem, as following:

$$\min \sum_{j=1}^{n}((\alpha_j^- + \alpha_j^+) + \sum_{w=1}^{p}(\beta_{wj}^- + \beta_{wj}^+))$$

$$s.t. \sum_{r=1}^{s} u_r y_{rj} + \sum_{w=1}^{p} g_{wj} - (\sum_{i=1}^{m} v_i x_{ij} + f_j) = 0, \forall j,$$

$$f_j + \alpha_j^- - \alpha_j^+ = v_{m+1}\lambda_j F, \forall j,$$

$$g_{wj} + \beta_{wj}^- - \beta_{wj}^+ = u_{s+w}\mu_j G_w, \forall j, w,$$

$$\sum_{j=1}^{n} f_j \;\; = v_{m+1}F,$$

$$\sum_{j=1}^{n} g_{wj} = \; u_{s+w}G_w, \forall w,$$

$$u_r, v_i, u_{s+w}, v_{m+1} \geq \varepsilon, f_j, g_{wj}, \alpha_j^-, \alpha_j^+, \beta_{wj}^-, \beta_{wj}^+ \geq 0, \forall r, i, k, w, j. \qquad (7)$$

Lotf., et al., (2013) has successfully proved that model (7) always exists a feasible solution so that it can be used to subsidy allocation and target setting module in an appropriate way.

After obtaining the optimal solution of model (7), one can plug them into $u_{s+w}\overline{g_{wj}} = g_{wj}$ and $v_{m+1}\overline{f_j} = f_j$ so as to identify the optimal subsidy allocation and target setting to each bus routes simultaneously. With new additional subsidy input and targets output, all the bus routes would turn to be efficient units, indicating the proposed process helps bus routes optimize resource configuration and find out the most appropriate plan of resource utilization.

5.4.3 Case Study

The case study uses the proposed Route-level Subsidy Allocation and Target Setting Model to assign the Third Bus Company's incentive-based subsidy as well as distributing the designed targets of two scenarios across 17 bus routes operated by the Third Bus Company.

Section 5.3 has contracted two efficiency-based targets for all five bus companies in Chongqing, which are "Basic Plan" and "Proactive Plan" respectively. The former one encourages the Third Bus Company to increase its ridership by 0.8 million and mileage by 1 million which contribute to a raise of 0.06845 and 0.09137 for financial and operational efficiency correspondingly. Those improvements further bring in about 12 million RMB (￥11970873) incentive-based subsidies to help operator achieve targets. Meanwhile, the latter plan inspires the Third Bus Company to generate an increment of 1 million for ridership and 1.2 million for mileage, accordingly, the financial and operational efficiency go up by 0.0799 and 0.09137 respectively, which make operator acquire about 17 million RMB (￥16844735). Furthermore, the proposed MOFP-based DEA model is firstly activated to measure the relative efficiency between input cost and operational output for each selected route before the subsidy allocation and target setting. The obtained value is further developed into a reference to compare the efficiency value improvement after route-level subsidy allocation and target assignment. Table 5.10 records the operational data of 17 bus routes in 2014.

Table 5.10 Data of 17 bus routes of 2014

Bus Routes	Operation Cost (Yuan)	Mileages (Vehicle*Km)	Passenger Volume (Trip)	MOFP-based DEA Efficiency
301	20504617	3421040	10626595	0.99491
308	9226176	1507086	4785091	0.99805
318	18286725	3045303	9468027	0.99604
319	13051944	2154064	6788329	0.99391
325	7562633	1292405	3906875	0.99317
338	10421869	1752891	5423317	0.99007
346	11503345	1905822	5995524	0.99147
349	14554268	2444178	7548693	0.99308
354	22040984	3650440	11459348	0.99358
362	13986874	2304464	7224364	0.99993
363	16659564	2762973	8671525	0.99238
364	20552895	3411246	10667675	0.99460
365	11669153	1918061	6029770	1.00000
368	7607840	1282784	3947236	0.99207
372	6344398	1050294	3273434	0.99996
381	8399102	1405540	4362770	0.99249
382	3684755	636254	1883323	1.00000

"Basic Plan" Scenario

In this scenario, additional input of subsidy F is defined to 11970873 while additional output of targets G_1 and G_2 for mileage and ridership increments are confirmed to 800,000 and 1,000,000 respectively. By adoption of proposed model, the subsidy allocation and target setting plan is demonstrated in Table 5.11 and Figure 5.13.

Table 5.11 Subsidy and target setting result of Basic Plan

Route	Subsidy	Ridership Target	Mileage Target	MOFP-based DEA Efficiency
301	1136079.9	+37964.9	+137537.2	1.00000
308	511186.0	+17005.1	+88098.3	1.00000
318	1013195.2	+135459.7	+42272.8	1.00000
319	723156.7	+24985.3	+69342.0	1.00000
325	419015.6	+37734.3	+17564.3	1.00000
338	583551.2	+19394.3	+24242.8	1.00000
346	637354.9	+27674.2	+26692.5	1.00000

349	806394.5	+64905.8	+33758.2	1.00000
354	1221203.9	+102518.0	+51044.3	1.00000
362	774957.5	+25752.4	+164511.9	1.00000
363	923040.6	+56594.8	+38628.3	1.00000
364	1138754.7	+118932.0	+47561.8	1.00000
365	646541.7	+21479.6	+137493.6	1.00000
368	415403.8	+20311.2	+17668.2	1.00000
372	351518.0	+11685.2	+75585.5	1.00000
381	465361.1	+31372.6	+19486.6	1.00000
382	204157.7	+46230.6	+8511.7	1.00000
Total	***11970873***	***800000***	***1000000***	

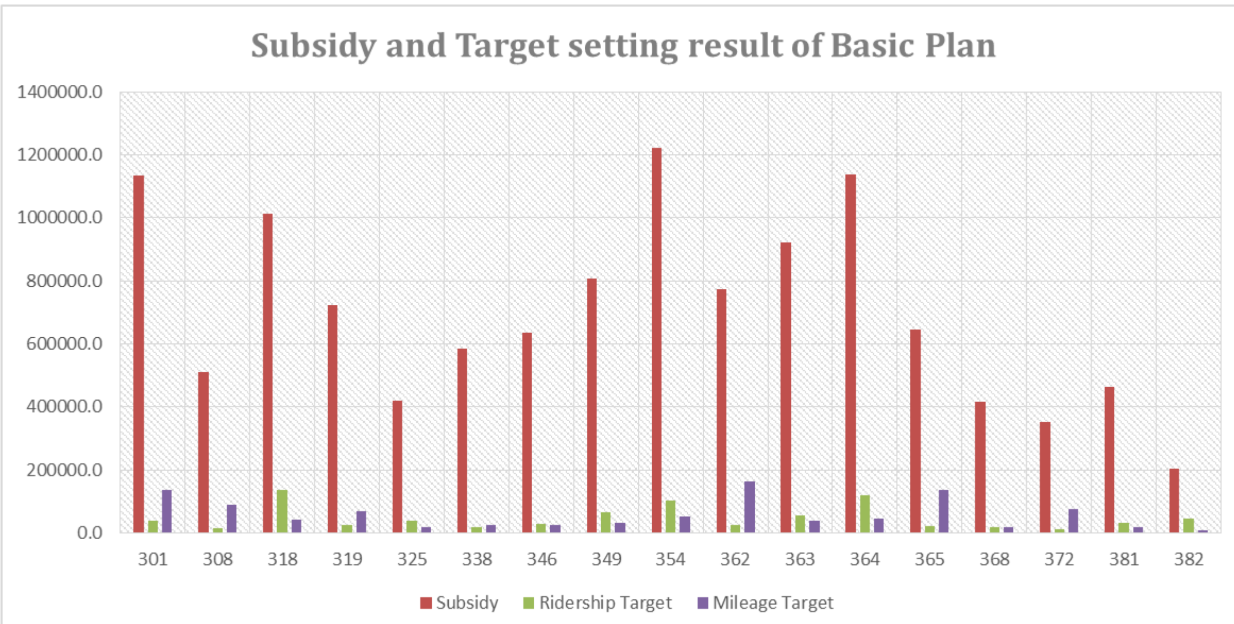


Figure 5.13 Subsidy and target setting result of Basic Plan

"Proactive Plan"

In proactive scenario, additional input of subsidy F increases to 16844735.09 while additional output of targets G_1 *and* G_2 for mileage and ridership accordingly jump to 1,000,000 and 1,200,000 respectively. By implementation of proposed model, the subsidy allocation and target setting plan is recorded in Table 5.12 and Figure 5.14.

Table 5.12 Subsidy and target setting result of Proactive Plan

Route	Subsidy	Ridership Target	Mileage Target	MOFP-based DEA Efficiency
301	1598627.3	+47456.1	+163753.7	1.00000
308	719312.0	+102462.6	+25507.7	1.00000
318	1425711.0	+42272.8	+168245.5	1.00000
319	1017585.1	+30209.4	+92392.0	1.00000
325	589615.1	+39811.7	+21950.2	1.00000
338	812533.3	+24242.8	+29091.4	1.00000
346	896849.8	+46067.7	+32031.0	1.00000
349	1134712.7	+80348.7	+40509.8	1.00000
354	1718409.0	+137272.3	+61253.1	1.00000
362	1090476.3	+32190.5	+187838.7	1.00000
363	1298850.6	+38628.3	+90057.2	1.00000
364	1602391.2	+149354.1	+57074.1	1.00000
365	909776.9	+26849.6	+157967.9	1.00000
368	593139.6	+33459.5	+21201.8	1.00000
372	494636.3	+82110.6	+17527.8	1.00000
381	654829.7	+41490.8	+23384.0	1.00000
382	287279.2	+45772.4	+10214.0	1.00000
Total	***16844735***	***1000000***	***1200000***	

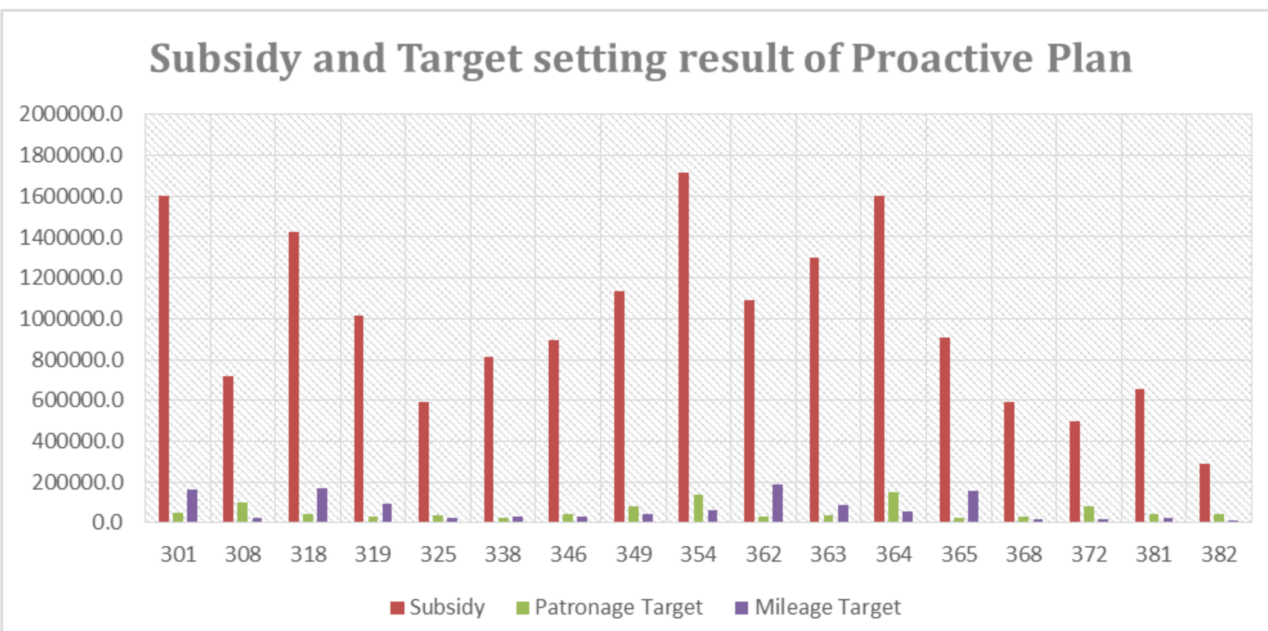


Figure 5.14 Subsidy and target setting result of Proactive Plan

By analysis on results, it can be concluded that the allocation of subsidies and determination of targets for each bus route are not only relied on resource utilization and the relationship between inputs and outputs but also giving a full consideration to their operational capabilities. As the larger operational scale, the more cost is required to maintain and improve the service and efficiency. Basically, the proposed methodology aims at optimizing system resource configuration according to each route's ability and strength. In detailed, Route 301, 318,354 and 364 are ranked in the first group which are assigned more subsidies than others while the targets of ridership and mileage increments are also relatively higher since those units have enough capabilities to cope with higher requirements. In the meantime, route 368, 372, 381 and 382 are distributed less subsidies and targets due to their limitation and bottleneck in operational capabilities. Another remarkable feature need to be highlighted that all the bus routes have reached to efficient frontier after adding the new subsidy additional input and additional output targets which indicates an optimal process on resource utilization brought by proposed model.

5.4.4 Conclusion

This section developed a MOFP-based DEA model to subdivide Bus Company's incentive-based subsidy to its managed bus routes as well as distributing Bus Company's targets into selected bus routes simultaneously. The proposed model borrows the concept of MOP programming, which is able to simultaneously optimize two or more conflicting objectives subject to certain constraints. The allocated subsidy now is considered as a new additional input while the set targets are treated as new additional outputs which forms an advanced system to aim at optimizing system resource utilization. The developed Route-level Subsidy Allocation and Target Setting Model factors in the following features:

- Simultaneous assignment on additional input and outputs;
- Fix resource allocation;
- Centralized resource allocation.

The designed model has been successfully linearized to locate global solution and proved that it always exists a feasible solution. As a natural extension of bus operator-level subsidy allocation study, 17 bus routes in Chongqing Third Bus Company is selected as a case study to share the incentive-based subsidy and set targets of ridership and mileage increases generated from **section 5.3**. The results from two different scenarios, "Basic Plan" and "Proactive Plan", show that each bus route is assigned with a reasonable level of subsidy and targets to help them reaching at the "efficient" status, which fully demonstrate the system's advantages over traditional methods.

Chapter 6: Summary and Conclusions

This dissertation develops a multi-dimensional framework consisting of a series of robust multi-criteria evaluation models to assess the operational and financial performance of transit systems at various levels of application. It further contributes to creating a close loop between transit efficiency evaluation and subsidy allocation by developing a set of incentive-based resource allocation models taking various levels of operational and financial efficiencies into consideration. Case studies using real-world transit data will be performed to validate the performance and applicability of the proposed models. In total, this dissertation has made several contributions in the following aspects:

6.1. Multi-dimensional Transit System Efficiency Assessment

This dissertation firstly develops an integrated framework with quantitative approaches for comprehensive multi-dimensional transit system efficiency assessment.

At the city/regional level, this study presents a multi-dimensional evaluation framework which contains the policy level and the technical level to compare the performance of different cities/regions in the development of public transport system. A two level Fuzzy-AHP model is developed to reflect the impacts from both policy and technical levels. The "policy level" is designed to capture a city's characteristics and developing priorities as well as the subjective opinions of various transit stakeholders during the evaluation process, while the "technical level" functions to compare and assess detailed technical indicators with an enhanced multi-criteria ranking model. The proposed model features the integration of the fuzzy logic with a hierarchical AHP structure to: 1) normalize the scales of different evaluation indicators, 2) construct the matrix of pair-wise comparisons with fuzzy set, 3) optimize the weight of each criterion with a non-linear programming model, and 4) synthesize the final score for evaluating the transit development levels. Consequently, the proposed framework offers the advantage of preventing the vagueness and uncertainty of the decision-maker(s) when evaluating technical criteria while properly retaining the policy preferences from decision makers. It selects nine cities in the Chongqing metropolitan area for a case study. Results reveal that the proposed evaluation framework and model can effectively generate the overall rankings of different cities/regions in transit system development and also identify microscopic deficiencies and areas of improvement for a city with respect to any specific criterion.

At the bus operator-level, this dissertation presents an enhanced Data Envelop Analysis (DEA) model which modifies conventional DEA model by adding the constraint cones generated from the Fuzzy-AHP model to evaluate transit operator's efficiency. The proposed model factors in: 1) solving a biased assumption of conventional DEA that no output or input is more important than the others, which features the integration of a Fuzzy-AHP model to generate cone constraints; 2) offering the advantages in breaking the tie between those efficient units under the conventional DEA. To illustrate the applicability of the proposed approach, a real case in Nanjing City, the capital of Jiangsu province has been has been selected where the efficiencies of seven bus companies are assessed based on 2009 and 2010 dataset. A comparison between conventional DEA and enhanced DEA is also unfolded to clarify the new system's dominance. Results reveal that the proposed model is more applicable in evaluating the transit operator's efficiency and encouraging a boarder range of applications.

At the bus route-level, this dissertation contributes to filling the vacancy of a Bootstrap-Super DEA model with sufficient capability to remedy the limitations: 1) a tie in efficient units, and 2) ignorance of statistical test. In proposed system, a super-DEA model is firstly designed to assess the bus routes' efficiency by which the theoretical defect of lacking capability to rank those efficient units in conventional DEA is efficiently solved. A following step, the Bootstrap method, is applied to modify the efficiency derived from super-DEA model as well as generating the efficiency distribution and taking statistical test into account. After the implementation of Bootstrap method, a corrected efficiency value and the corresponding confidence interval are offered. The obtained interval is further considered as the benchmark and reference for manager to monitor and control the transit operation. To illustrate the usefulness and usability of the approach, a real case in Chongqing Metropolitan, China has been summarized to evaluate 17 bus routes' efficiency. A comparison between conventional DEA, Super-DEA and Bootstrap Super-DEA with detailed discussions is unfolded to clarify the new model's functions. Results reveal that the proposed model is more applicable in evaluating the transit operator's efficiency and encouraging a boarder range of applications.

6.2. Incentive-based Subsidy Allocation

After a detailed analysis on transit system performance evaluation, this study moves the focus to design an incentive-based subsidy allocation mechanism at bus operator-level and bus route-level. Furthermore, this dissertation also demonstrates some efforts in developing the appropriate and functional models to cope with the specific requirements at different levels.

At the bus operator-level, it contributes to filling the vacancy of a theoretically justified model in literature that can allocate limited subsidies to urban transit operators according to their operational and financial efficiencies. A comprehensive evaluation and decision framework is developed, consisting of key modules of baseline assessment of transit operational and financial efficiency, efficiency-based target setting and pre-evaluation, incentive-based subsidy allocation, as well as feedback and subsidy adjustment. When evaluating the operational efficiency of transit operators, the proposed Constrained Cone-based Enhanced DEA developed is activated. Noticeably, when evaluating the financial efficiency of transit operators, a revised SBM Super Efficiency Model is developed to directly account for input cost and output slack in efficiency measurements, with the advantage of capturing the whole aspect of inefficiency. Using the obtained financial and operational efficiencies as the new set of outputs and the operational cost as the new set of inputs, an extended inverse DEA model is developed to allocate incentive-based subsidy. The model allows each transit operator to set the target output and efficiency level according to its operational constraints and capabilities. The corresponding improvements are then processed by the inverse DEA model to identify extra inputs to each transit operator (i.e. the subsidy allocated). Therefore, allocation of the subsidy mainly depends on the improvement of a transit operator's performance rather than the running cost.

The proposed model has selected Chongqing Municipality, China as a case study for subsidy allocation of five transit operators regulated by the municipal government. Results demonstrate an advantage of proposed strategy over traditional framework.

At the bus route-level, this dissertation develops a MOFP-based model to prioritize a bus company's incentive-based subsidy to its managed bus routes as well as distributing a bus company's targets into selected bus routes simultaneously. The proposed model is developed based on the concept of MOP programming, which is able to simultaneously optimize two or more conflicting objectives subject to certain constraints. The allocated subsidy now is considered as a new additional input while the set targets are treated as new additional outputs which forms an advanced system to aim at optimizing system resource utilization.

The designed model has been successfully linearized to obtain globally optimal solution. As a natural extension of bus operator-level subsidy allocation study, 17 bus routes in Chongqing Third Bus Company are selected as a case study to share the incentive-based subsidy and set targets of ridership and mileage increases. The results from two different scenarios, “Basic Plan” and “Proactive Plan”, show that each bus route is assigned with a reasonable level of subsidy and targets to further help them reaching the “efficient” status, which fully demonstrate the system’s advantages over traditional methods.

6.3 Future Research

In terms of transit system performance evaluation, future research may focus on introducing an artificial intelligence package to pre-process and identify the attributes of original data which could assign the proposed model or framework an ability to classify data into different layers. In the meantime, the joint interface to accommodate transit big data, likely, GPS data, AFC data is also recommended. Furthermore, a module of post-assessment is necessary to fine-tune the proposed model.

In terms of incentive-based subsidy allocation, future research may attempt to employ the game theory to reflect the conflicts and cooperation among government, bus operators and passengers during the process of subsidy allocation.

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